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CP Violation - A New Era

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Abstract

We give a pedagogical review of the theory of CP violation with emphasis on the implications of recent experimental results. The review includes: (i) A detailed description of how CP violation arises in the Standard Model and in its extension that allows for neutrino masses; (ii) The formalism of CP violation in meson decays and its application to various K decays (ε_K, ε'/ε and $K \to \pi\nu\nu$), D decays ($D \to K\pi$ and $D \to KK$) and B decays ($B \to \ell\nu X$, $B \to \psi K_S$ and $B \to \pi\pi$, including a discussion of the 'penguin pollution' problem); (iii) Supersymmetry: the CP problems and the use of CP violation as a probe of the mechanism of dynamical supersymmetry breaking.

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Contents

I. INTRODUCTION

The Standard Model predicts that the only way that CP is violated is through the Kobayashi-Maskawa mechanism [1]. Specifically, the source of CP violation is a *single* phase in the mixing matrix that describes the charged current weak interactions of quarks.

In the introductory chapter, we briefly review the present evidence that supports the Kobayashi-Maskawa picture of CP violation, as well as the various arguments against this picture.

A. Why Believe the Kobayashi-Maskawa Mechanism?

Experiments have measured to date three independent CP violating observables:
(i) Indirect CP violation in $K \to \pi\pi$ decays [2] and in $K \to \pi\ell\nu$ decays is given by:

$$\varepsilon_K = (2.28 \pm 0.02) \times 10^{-3}\ e^{i\pi/4}. \tag{1.1}$$

(ii) Direct CP violation in $K \to \pi\pi$ decays [3–7] is given by

$$\frac{\varepsilon'}{\varepsilon} = (1.72 \pm 0.18) \times 10^{-3}. \tag{1.2}$$

(iii) CP violation in $B \to \psi K_S$ decay and other, related modes has been measured [8–12]:

$$a_{\psi K_S} = 0.79 \pm 0.10. \tag{1.3}$$

All three measurements are consistent with the Kobayashi-Maskawa picture of CP violation. In particular, the two recent measurements of CP violation in B decays [11,12] have provided the first precision test of CP violation in the Standard Model. Since the model has passed this test successfully, we are able, for the first time, to make the following statement: *The Kobayashi-Maskawa phase is, very likely, the dominant source of CP violation in low-energy flavor-changing processes.*

In contrast, various alternative scenarios of CP violation that have been phenomenologically viable for many years are now unambiguously excluded. Two important examples are the following:

1. The superweak framework [13], that is, the idea that CP violation is purely indirect, is excluded by the evidence that $\varepsilon'/\varepsilon \neq 0$.

2. Approximate CP, that is, the idea that all CP violating phases are small, is excluded by the evidence that $a_{\psi K_S} = \mathcal{O}(1)$.

The experimental result (1.3) and its implications for theory signify a new era in the study of CP violation. In this series of lectures we will explain these recent developements and their significance.

B. Why Doubt the Kobayashi-Maskawa Mechanism?

Cosmology shows that the Kobayashi-Maskawa phase cannot be the only source of CP violation: baryogenesis, that is, the history of matter and antimatter in the Universe, cannot be accounted for by the Kobayashi-Maskawa mechanism.

To understand this statement, let us provisionally switch off all CP violation. Then, for every process that occurs in Nature, the corresponding CP conjugate process proceeds with a precisely equal rate. Let us further assume that the initial conditions are such that the number density of quarks and the number density of the matching antiquarks are equal. Then, CP invariance guarantees that the number densities remain equal to each other along the history of the Universe. In other words, the baryon asymmetry, $\eta \equiv (n_B - n_{\bar{B}})/n_\gamma$, is guaranteed to remain zero. Two particularly significant processes are proton-antiproton annihilation and production. While the first would happen at any temperature, the latter is allowed only if the energy of the photons is large enough to produce a proton-antiproton pair. At high enough termperatures, $T \gtrsim 2m_p$, annihilation and production will keep the protons and antiprotons in equilibrium and their number densities would be (precisely equal to each other and) similar to the photon number density, $n_B = n_{\bar{B}} \approx n_\gamma$. But at temperatures well below GeV, proton-antiproton production slows down until it practically stops. Since annihilation continues to take place, the number densities of protons and antiprotons (remain equal to each other but) decrease, and at present there would be practically neither matter nor antimatter. This is, of course, inconsistent with observations.

Now let us switch on CP violation. That allows a different rate for a process and its CP conjugate. Such a situation would have relevant consequences if two more conditions are met [14]: there is a departure from thermal equilibrium and baryon number can be violated. When all three conditions are satisfied, a difference between the number densities of quarks and of antiquarks can be induced. We assume that the number of quarks becomes slightly larger than the number of antiquarks. This scenario is called *baryogenesis*. At the electroweak phase transition (temperatures of order a few hundred GeV, $t \sim 10^{-11}$ seconds) baryon number violating processes become highly suppressed, and the baryon number cannot change any longer. The history of matter and antimatter in the Universe proceeds along the same lines as described in the previous paragraph. In particular, at temperatures well below GeV the number densities of protons and antiprotons decrease. There is however an important difference: at some time, practically all antiprotons would disappear. But the small surplus of protons have no matching antiprotons to annihilate with. It remains there forever. The resulting picture of the present Universe is then as follows: there is no antimatter. There is a small amount of matter, with the present ratio $(n_B/n_\gamma)_0$ reflecting the baryon asymmetry, $[(n_q - n_{\bar{q}})/n_\gamma]_{\rm BG}$, induced by baryogenesis. This picture is qualitatively consistent with observations. Thus we have good reasons to think that we understand the general mechanism of baryogenesis.

The important point for our purposes is that baryogenesis is a consequence of CP violating processes. Therefore the present baryon number, which is accurately deduced from nucleosynthesis constraints (for a recent analysis, see [15]),

$$\frac{n_B}{n_\gamma} = (5.5 \pm 0.5) \times 10^{-10}, \tag{1.4}$$

is essentially a CP violating observable! It can be added to the list of known CP violating observables, eqs. (1.1), (1.2) and (1.3). Within a given model of CP violation, one can check for consistency between the data from cosmology, eq. (1.4), and those from laboratory experiments.

The surprising point is that the Kobayashi-Maskawa mechanism for CP violation fails to account for (1.4). It predicts present baryon number density that is many orders of magnitude below the observed value [16–18]. This failure is independent of other aspects of the Standard Model: the suppression of n_B/n_γ from CP violation is much too strong, even if the departure from thermal equilibrium is induced by mechanisms beyond the Standard Model. This situation allows us to make the following statement: *There must exist sources of CP violation beyond the Kobayashi-Maskawa phase.*

Three important examples of viable models of baryogenesis are the following:

1. GUT baryogenesis (for a recent review see [19]): the source of the baryon asymmetry is in CP violating decays of heavy bosons related to grand unified theories. In general, baryon number is not a conserved quantity in GUTs. Departure from thermal equilibrium is provided if the lifetime of the heavy boson is long enough that it decays when the temperature is well below its mass. The relevant CP violating parameters are not expected to affect low energy observables.

2. Leptogenesis (for a recent review see [20]): lepton asymmetry is induced by CP violating decays of heavy fermions that are singlets of the Standard Model gauge group (sterile neutrinos). Departure from thermal equilibrium is provided if the lifetime of the heavy neutrino is long enough that it decays when the temperature is below its mass. $B+L$-violating processes are fast before the electroweak phase transition and convert the lepton asymmetry into a baryon asymmetry. The CP violating parameters may be related to CP violation in the mixing matrix for the light neutrinos (but this is a model dependent issue [21]).

3. Electroweak baryogenesis (for a review see [22]): the source of baryon asymmetry is the interactions of top (anti)quarks with the Higgs field during the electroweak phase transition. CP violation is induced, for example, by supersymmetric interactions. Sphaleron configurations provide baryon number violating interactions. Departure from thermal equilibrium is provided by the wall between the false vacuum ($\langle\phi\rangle = 0$) and the expanding bubble with the true vacuum, where electroweak symmetry is broken.

2. The Strong CP Problem

Nonperturbative QCD effects induce an additional term in the SM Lagrangian,

$$\mathcal{L}_\theta = \frac{\theta_{\rm QCD}}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu a}F^{\rho\sigma a}. \tag{1.5}$$

This term violates CP. In particular, it induces an electric dipole moment (EDM) to the neutron. The leading contribution in the chiral limit is given by [23]

$$\begin{aligned} d_N &= \frac{g_{\pi NN}\bar{g}_{\pi NN}}{4\pi^2 M_N}\ln(M_N/m_\pi) \\ &\approx 5\times 10^{-16}\ \theta_{\rm QCD}\ e\ {\rm cm}, \end{aligned} \tag{1.6}$$

where M_N is the nucleon mass, and $g_{\pi NN}$ ($\bar{g}_{\pi NN}$) is the pseudoscalar coupling (CP-violating scalar coupling) of the pion to the nucleon. (The leading contribution in the large N_c limit was calculated in the Skyrme model [24] and leads to a similar estimate.) The experimental bound on d_N is given by

$$d_N \leq 6.3 \times 10^{-26}\ e\ \text{cm} \quad [25]. \tag{1.7}$$

It leads to the following bound on $\theta_{\rm QCD}$:

$$\theta_{\rm QCD} \lesssim 10^{-10}. \tag{1.8}$$

Since $\theta_{\rm QCD}$ arises from nonperturbative QCD effects, it is impossible to calculate it. Yet, there are good reasons to expect that these effects should yield $\theta_{\rm QCD} = \mathcal{O}(1)$ (for a clear review of this subject, see [26]). Within the SM, a value as small as (1.8) is unnatural, since setting $\theta_{\rm QCD}$ to zero does not add symmetry to the model. [In particular, as we will see below, CP is violated by $\delta_{\rm KM} = \mathcal{O}(1)$.] Understanding why CP is so small in the strong interactions is the strong CP problem.

It seems then that the strong CP problem is a clue to new physics. Among the solutions that have been proposed are a massless u-quark (for a review, see [27]), the Peccei-Quinn mechanism [28,29] and spontaneous CP violation. As concerns the latter, it is interesting to note that in various string theory compactifications, CP is an exact gauge symmetry and must be spontaneously broken [30,31].

3. New Physics

Another motivation to measure CP violating processes is that almost any extension of the Standard Model provides new sources of CP violation. These sources often allow for significant deviations from the Standard Model predictions. Moreover, various CP violating observables can be calculated with very small hadronic uncertainties. Consequently, CP violation provides an excellent probe of new physics.

C. Will New CP Violation Be Observed In Experiments?

The SM picture of CP violation is testable because the Kobayashi-Maskawa mechanism is unique and predictive. These features are mainly related to the fact that there is a single phase that is responsible to all CP violation. As a consequence of this situation, one finds two classes of tests:

(i) Correlations: many independent CP violating observables are correlated within the SM. For example, the SM predicts that the CP asymmetries in $B \to \psi K_S$ and in $B \to \phi K_S$, which proceed through different quark decay processes, are equal to each other. Another important example is the strong SM correlation between CP violation in $B \to \psi K_S$ and in $K \to \pi \nu \bar{\nu}$.

(ii) Zeros: since the KM phase appears in flavor-changing, weak-interaction couplings of quarks, and only if all three generations are involved, many CP violating observables are predicted to be negligibly small. For example, the SM predicts no CP violation in the lepton sector, practically no CP violation in flavor-diagonal processes (*i.e.* a tiny electric dipole moment for the neutron) and very small CP violation in tree level D decays.

In addition, several CP violating observables can be calculated with very small hadronic uncertainties.

The strongest argument that new sources of CP violation must exist in Nature comes from baryogenesis. Whether the CP violation that is responsible for baryogenesis would be manifest in measurements of CP asymmetries in B decays depends on two issues:

(i) The scale of the new CP violation: if the relevant scale is very high, such as in GUT baryogenesis or leptogenesis, the effects cannot be signalled in these measurements. To estimate the limit on the scale, the following three facts are relevant: First, the Standard Model contributions to CP asymmetries in B decays are $\mathcal{O}(1)$. Second, the expected experimental accuracy would reach in some cases the few percent level. Third, the contributions from new physics are expected to be suppressed by $(\Lambda_{\rm EW}/\Lambda_{\rm NP})^2$. The conclusion is that, if the new source of CP violation is related to physics at $\Lambda_{\rm NP} \gg 1\ TeV$, it cannot be signalled in B decays. Only if the true mechanism is electroweak baryogenesis, it can potentially affect B decays.

(ii) The flavor dependence of the new CP violation: if it is flavor diagonal, its effects on B decays would be highly suppressed. It can still manifest itself in other, flavor diagonal CP violating observables, such as electric dipole moments.

We conclude that new measurements of CP asymmetries in meson decays are particularly sensitive to new sources of CP violation that come from physics at (or below) the few TeV scale and that are related to flavor changing couplings. This is, for example, the case, in certain supersymmetric models of baryogenesis [32,33]. The search for electric dipole moments can reveal the existence of new flavor diagonal CP violation.

Of course, there could be new flavor physics at the TeV scale that is not related to the baryon asymmetry and may give signals in B decays. The best motivated extension of the SM where this situation is likely is that of supersymmetry. We will discuss supersymmetric CP violation in the last chapter.

II. THE KOBAYASHI-MASKAWA MECHANISM

A. Yukawa Interactions are the Source of CP Violation

A model of elementary particles and their interactions is defined by three ingredients:
(i) The symmetries of the Lagrangian;
(ii) The representations of fermions and scalars;
(iii) The pattern of spontaneous symmetry breaking.
The Standard Model (SM) is defined as follows:
(i) The gauge symmetry is

$$G_{\rm SM} = SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}. \tag{2.1}$$

(ii) There are three fermion generations, each consisting of five representations of $G_{\rm SM}$:

$$Q^I_{Li}(3,2)_{+1/6}, \quad U^I_{Ri}(3,1)_{+2/3}, \quad D^I_{Ri}(3,1)_{-1/3}, \quad L^I_{Li}(1,2)_{-1/2}, \quad E^I_{Ri}(1,1)_{-1}. \tag{2.2}$$

Our notations mean that, for example, left-handed quarks, Q_L^I, are triplets of $SU(3)_{\rm C}$, doublets of $SU(2)_{\rm L}$ and carry hypercharge $Y=+1/6$. The super-index I denotes interaction eigenstates. The sub-index $i=1,2,3$ is the flavor (or generation) index.

There is a single scalar representation,

$$\phi(1,2)_{+1/2}. \tag{2.3}$$

(iii) The scalar ϕ assumes a VEV,

$$\langle\phi\rangle=\begin{pmatrix}0\\ \frac{v}{\sqrt{2}}\end{pmatrix}, \tag{2.4}$$

so that the gauge group is spontaneously broken,

$$G_{\rm SM}\to SU(3)_{\rm C}\times U(1)_{\rm EM}. \tag{2.5}$$

The Standard Model Lagrangian, $\mathcal{L}_{\rm SM}$, is the most general renormalizable Lagrangian that is consistent with the gauge symmetry (2.1). It can be divided to three parts:

$$\mathcal{L}_{\rm SM}=\mathcal{L}_{\rm kinetic}+\mathcal{L}_{\rm Higgs}+\mathcal{L}_{\rm Yukawa}. \tag{2.6}$$

As concerns the kinetic terms, to maintain gauge invariance, one has to replace the derivative with a covariant derivative:

$$D^\mu=\partial^\mu+ig_sG_a^\mu L_a+igW_b^\mu T_b+ig'B^\mu Y. \tag{2.7}$$

Here G_a^μ are the eight gluon fields, W_b^μ the three weak interaction bosons and B^μ the single hypercharge boson. The L_a's are $SU(3)_{\rm C}$ generators (the 3×3 Gell-Mann matrices $\frac{1}{2}\lambda_a$ for triplets, 0 for singlets), the T_b's are $SU(2)_{\rm L}$ generators (the 2×2 Pauli matrices $\frac{1}{2}\tau_b$ for doublets, 0 for singlets), and Y are the $U(1)_{\rm Y}$ charges. For example, for the left-handed quarks Q_L^I, we have

$$\mathcal{L}_{\rm kinetic}(Q_L)=i\overline{Q_{Li}^I}\gamma_\mu\left(\partial^\mu+\frac{i}{2}g_sG_a^\mu\lambda_a+\frac{i}{2}gW_b^\mu\tau_b+\frac{i}{6}g'B^\mu\right)Q_{Li}^I, \tag{2.8}$$

while for the left-handed leptons L_L^I, we have

$$\mathcal{L}_{\rm kinetic}(L_L)=i\overline{L_{Li}^I}\gamma_\mu\left(\partial^\mu+\frac{i}{2}gW_b^\mu\tau_b-ig'B^\mu\right)L_{Li}^I. \tag{2.9}$$

These parts of the interaction Lagrangian are always CP conserving.

The Higgs potential, which describes the scalar self interactions, is given by:

$$\mathcal{L}_{\rm Higgs}=\mu^2\phi^\dagger\phi-\lambda(\phi^\dagger\phi)^2. \tag{2.10}$$

For the Standard Model scalar sector, where there is a single doublet, this part of the Lagrangian is also CP conserving. For extended scalar sector, such as that of a two Higgs doublet model, $\mathcal{L}_{\rm Higgs}$ can be CP violating. Even in case that it is CP symmetric, it may lead to spontaneous CP violation.

The quark Yukawa interactions are given by

$$-\mathcal{L}_{\rm Yukawa}^{\rm quarks}=Y_{ij}^d\overline{Q_{Li}^I}\phi D_{Rj}^I+Y_{ij}^u\overline{Q_{Li}^I}\tilde{\phi}U_{Rj}^I+{\rm h.c.}. \tag{2.11}$$

This part of the Lagrangian is, in general, CP violating. More precisely, CP is violated if and only if [34]

$${\rm Im}\left\{\det[Y^dY^{d\dagger},Y^uY^{u\dagger}]\right\}\neq 0. \tag{2.12}$$

An intuitive explanation of why CP violation is related to *complex* Yukawa couplings goes as follows. The hermiticity of the Lagrangian implies that $\mathcal{L}_{\rm Yukawa}$ has its terms in pairs of the form

$$Y_{ij}\overline{\psi_{Li}}\phi\psi_{Rj}+Y_{ij}^*\overline{\psi_{Rj}}\phi^\dagger\psi_{Li}. \tag{2.13}$$

A CP transformation exchanges the operators

$$\overline{\psi_{Li}}\phi\psi_{Rj}\leftrightarrow\overline{\psi_{Rj}}\phi^\dagger\psi_{Li}, \tag{2.14}$$

but leaves their coefficients, Y_{ij} and Y_{ij}^*, unchanged. This means that CP is a symmetry of $\mathcal{L}_{\rm Yukawa}$ if $Y_{ij}=Y_{ij}^*$.

The lepton Yukawa interactions are given by

$$-\mathcal{L}_{\rm Yukawa}^{\rm leptons}=Y_{ij}^e\overline{L_{Li}^I}\phi E_{Rj}^I+{\rm h.c.}. \tag{2.15}$$

It leads, as we will see in the next section, to charged lepton masses but predicts massless neutrinos. Recent measurements of the fluxes of atmospheric and solar neutrinos provide evidence for neutrino masses. That means that $\mathcal{L}_{\rm SM}$ cannot be a complete description of Nature. The simplest way to allow for neutrino masses is to add dimension-five (and, therefore, nonrenormalizable) terms, consistent with the SM symmetry and particle content:

$$-\mathcal{L}_{\rm Yukawa}^{\rm dim-5}=\frac{Y_{ij}^\nu}{M}L_iL_j\phi\phi+{\rm h.c.}. \tag{2.16}$$

The parameter M has dimension of mass. The dimensionless couplings Y_{ij}^ν are symmetric ($Y_{ij}^\nu=Y_{ji}^\nu$). We will refer to the SM extended to include the terms $\mathcal{L}_{\rm Yukawa}^{\rm dim-5}$ of eq. (2.16) as the "extended SM" (ESM):

$$\mathcal{L}_{\rm ESM}=\mathcal{L}_{\rm kinetic}+\mathcal{L}_{\rm Higgs}+\mathcal{L}_{\rm Yukawa}+\mathcal{L}_{\rm Yukawa}^{\rm dim-5}. \tag{2.17}$$

The inclusion of nonrenormalizable terms is equivalent to postulating that the SM is only a low energy effective theory, and that new physics appears at the scale M.

How many independent CP violating parameters are there in $\mathcal{L}_{\rm Yukawa}^{\rm quarks}$? Each of the two Yukawa matrices Y^q ($q=u,d$) is 3×3 and complex. Consequently, there are 18 real and 18 imaginary parameters in these matrices. Not all of them are, however, physical. One can think of the quark Yukawa couplings as spurions that break a global symmetry,

$$U(3)_Q\times U(3)_D\times U(3)_U\ \rightarrow\ U(1)_B. \tag{2.18}$$

This means that there is freedom to remove 9 real and 17 imaginary parameters [the number of parameters in three 3×3 unitary matrices minus the phase related to $U(1)_B$]. We conclude that there are 10 quark flavor parameters: 9 real ones and a single phase. This single phase is the source of CP violation in the quark sector.

How many independent CP violating parameters are there in the lepton Yukawa interactions? The matrix Y^e is a general complex 3×3 matrix and depends, therefore, on 9 real and 9 imaginary parameters. The matrix Y^ν is symmetric and depends on 6 real and 6 imaginary parameters. Not all of these 15 real and 15 imaginary parameters are physical. One can think of the lepton Yukawa couplings as spurions that break (completely) a global symmetry,

$$U(3)_L \times U(3)_E. \tag{2.19}$$

This means that 6 real and 12 imaginary parameters are not physical. We conclude that there are 12 lepton flavor parameters: 9 real ones and three phases. These three phases induce CP violation in the lepton sector.

B. CKM Mixing is the (Only!) Source of CP Violation in the Quark Sector

Upon the replacement $\mathrm{Re}(\phi^0) \to (v + H^0)/\sqrt{2}$ [see eq. (2.4)], the Yukawa interactions (2.11) give rise to mass terms:

$$-\mathcal{L}_M^q = (M_d)_{ij}\overline{D_{Li}^I}D_{Rj}^I + (M_u)_{ij}\overline{U_{Li}^I}U_{Rj}^I + \text{h.c.}, \tag{2.20}$$

where

$$M_q = \frac{v}{\sqrt{2}}Y^q, \tag{2.21}$$

and we decomposed the $SU(2)_\mathrm{L}$ quark doublets into their components:

$$Q_{Li}^I = \begin{pmatrix} U_{Li}^I \\ D_{Li}^I \end{pmatrix}. \tag{2.22}$$

The mass basis corresponds, by definition, to diagonal mass matrices. We can always find unitary matrices V_{qL} and V_{qR} such that

$$V_{qL}M_qV_{qR}^\dagger = M_q^{\mathrm{diag}} \quad (q = u, d), \tag{2.23}$$

with M_q^{diag} diagonal and real. The quark mass eigenstates are then identified as

$$q_{Li} = (V_{qL})_{ij}q_{Lj}^I, \quad q_{Ri} = (V_{qR})_{ij}q_{Rj}^I \quad (q = u, d). \tag{2.24}$$

The charged current interactions for quarks [that is the interactions of the charged $SU(2)_\mathrm{L}$ gauge bosons $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$], which in the interaction basis are described by (2.8), have a complicated form in the mass basis:

$$-\mathcal{L}_{W^\pm}^q = \frac{g}{\sqrt{2}}\overline{u_{Li}}\gamma^\mu(V_{uL}V_{dL}^\dagger)_{ij}d_{Lj}W_\mu^+ + \text{h.c.}. \tag{2.25}$$

The unitary 3×3 matrix,

$$V_{\rm CKM} = V_{uL} V_{dL}^\dagger, \quad (V_{\rm CKM} V_{\rm CKM}^\dagger = 1), \tag{2.26}$$

is the Cabibbo-Kobayashi-Maskawa (CKM) *mixing matrix* for quarks [35,1]. A unitary 3×3 matrix depends on nine parameters: three real angles and six phases.

The form of the matrix is not unique:

(i) There is freedom in defining $V_{\rm CKM}$ in that we can permute between the various generations. This freedom is fixed by ordering the up quarks and the down quarks by their masses, *i.e.* $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$. The elements of $V_{\rm CKM}$ are written as follows:

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{2.27}$$

(ii) There is further freedom in the phase structure of $V_{\rm CKM}$. Let us define P_q $(q = u, d)$ to be diagonal unitary (phase) matrices. Then, if instead of using V_{qL} and V_{qR} for the rotation (2.24) to the mass basis we use $\tilde{V}_{qL}$ and $\tilde{V}_{qR}$, defined by $\tilde{V}_{qL} = P_q V_{qL}$ and $\tilde{V}_{qR} = P_q V_{qR}$, we still maintain a legitimate mass basis since $M_q^{\rm diag}$ remains unchanged by such transformations. However, $V_{\rm CKM}$ does change:

$$V_{\rm CKM} \to P_u V_{\rm CKM} P_d^*. \tag{2.28}$$

This freedom is fixed by demanding that $V_{\rm CKM}$ has the minimal number of phases. In the three generation case $V_{\rm CKM}$ has a single phase. (There are five phase differences between the elements of P_u and P_d and, therefore, five of the six phases in the CKM matrix can be removed.) This is the Kobayashi-Maskawa phase $\delta_{\rm KM}$ which is the single source of CP violation in the quark sector of the Standard Model [1].

As a result of the fact that $V_{\rm CKM}$ is not diagonal, the $W^\pm$ gauge bosons couple to quark (mass eigenstates) of different generations. Within the Standard Model, this is the only source of *flavor changing* quark interactions.

C. The Three Phases in the MNS Mixing Matrix

The leptonic Yukawa interactions (2.15) and (2.16) give rise to mass terms:

$$-\mathcal{L}_M^\ell = (M_e)_{ij} \overline{e_{Li}^I} e_{Rj}^I + (M_\nu)_{ij} \nu_{Li}^I \nu_{Lj}^I + {\rm h.c.}, \tag{2.29}$$

where

$$M_e = \frac{v}{\sqrt{2}} Y^e, \quad M_\nu = \frac{v^2}{2M} Y^\nu, \tag{2.30}$$

and we decomposed the $SU(2)_{\rm L}$ lepton doublets into their components:

$$L_{Li}^I = \begin{pmatrix} \nu_{Li}^I \\ e_{Li}^I \end{pmatrix}. \tag{2.31}$$

We can always find unitary matrices V_{eL} and V_ν such that

$$V_{eL} M_e M_e^\dagger V_{eL}^\dagger = \text{diag}(m_e^2, m_\mu^2, m_\tau^2), \quad V_\nu M_\nu^\dagger M_\nu V_\nu^\dagger = \text{diag}(m_1^2, m_1^2, m_3^2). \tag{2.32}$$

The charged current interactions for leptons, which in the interaction basis are described by (2.9), have the following form in the mass basis:

$$-\mathcal{L}_{W^\pm}^\ell = \frac{g}{\sqrt{2}} \overline{e_{Li}} \gamma^\mu (V_{eL} V_\nu^\dagger)_{ij} \nu_{Lj} W_\mu^- + \text{h.c.}. \tag{2.33}$$

The unitary 3×3 matrix,

$$V_{\text{MNS}} = V_{eL} V_\nu^\dagger, \tag{2.34}$$

is the Maki-Nakagawa-Sakata (MNS) *mixing matrix* for leptons [36]. Similarly to the CKM matrix, the form of the MNS matrix is not unique. But there are differences in choosing conventions:

(*i*) We can permute between the various generations. This freedom is usually fixed in the following way. We order the charged leptons by their masses, *i.e.* $(e_1, e_2, e_3) \to (e, \mu, \tau)$. As concerns the neutrinos, one takes into account that the interpretation of atmospheric and solar neutrino data in terms of two-neutrino oscillations implies that $\Delta m^2_{\text{ATM}} \gg \Delta m^2_{\text{SOL}}$. It follows that one of the neutrino mass eigenstates is separated in its mass from the other two, which have a smaller mass difference. The convention is to denote this separated state by ν_3. For the remaining two neutrinos, ν_1 and ν_2, the convention is to call the heavier state ν_2. In other words, the three mass eigenstates are defined by the following conventions:

$$|\Delta m^2_{3i}| \gg |\Delta m^2_{21}|, \quad \Delta m^2_{21} > 0. \tag{2.35}$$

Note in particular that ν_3 can be either heavier or lighter than $\nu_{1,2}$. The elements of V_{MNS} are written as follows:

$$V_{\text{MNS}} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix}. \tag{2.36}$$

(*ii*) There is further freedom in the phase structure of V_{MNS}. (In the MNS paper [36] there is no reference to CP violation.) One can change the charged lepton mass basis by the transformation $e_{(L,R)i} \to e'_{(L,R)i} = (P_e)_{ii} e_{(L,R)i}$, where P_e is a phase matrix. There is, however, no similar freedom to redefine the neutrino mass eigenstates: From eq. (2.29) one learns that a transformation $\nu_L \to P_\nu \nu_L$ will introduce phases into the diagonal mass matrix. This is related to the Majorana nature of neutrino masses, assumed in eq. (2.16). The allowed transformation modifies V_{MNS}:

$$V_{\text{MNS}} \to P_e V_{\text{MNS}}. \tag{2.37}$$

This freedom is fixed by demanding that V_{MNS} will have the minimal number of phases. Out of six phases of a generic unitary 3×3 matrix, the multiplication by P_e can be used to remove three. We conclude that the three generation V_{MNS} matrix has three phases. One of these is the analog of the Kobayashi-Maskawa phase. It is the only source of CP violation in processes that conserve lepton number, such as neutrino flavor oscillations. The other two phases can affect lepton number changing processes.

With $V_{\text{MNS}} \neq \mathbf{1}$, the $W^\pm$ gauge bosons couple to lepton (mass eigenstates) of different generations. Within the ESM, this is the only source of *flavor changing* lepton interactions.

D. The Flavor Parameters

Examining the quark mass basis, one can easily identify the flavor parameters. In the quark sector, we have six quark masses and four mixing parameters: three mixing angles and a single phase.

The fact that there are only three real and one imaginary physical parameters in $V_{\rm CKM}$ can be made manifest by choosing an explicit parameterization. For example, the standard parameterization [37], used by the particle data group, is given by

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2.38}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The three $\sin\theta_{ij}$ are the three real mixing parameters while δ is the Kobayashi-Maskawa phase. Another, very useful, example is the Wolfenstein parametrization, where the four mixing parameters are (λ, A, ρ, η) with $\lambda = |V_{us}| = 0.22$ playing the role of an expansion parameter and η representing the CP violating phase [38]:

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \tag{2.39}$$

Various parametrizations differ in the way that the freedom of phase rotation, eq. (2.28), is used to leave a single phase in $V_{\rm CKM}$. One can define, however, a CP violating quantity in $V_{\rm CKM}$ that is independent of the parametrization [34]. This quantity, $J_{\rm CKM}$, is defined through

$$\mathrm{Im}[V_{ij}V_{kl}V_{il}^*V_{kj}^*] = J_{\rm CKM}\sum_{m,n=1}^{3}\epsilon_{ikm}\epsilon_{jln}, \quad (i,j,k,l = 1,2,3). \tag{2.40}$$

In terms of the explicit parametrizations given above, we have

$$J_{\rm CKM} = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta \simeq \lambda^6 A^2 \eta. \tag{2.41}$$

It is interesting to translate the condition (2.12) to the language of the flavor parameters in the mass basis. One finds that the following is a necessary and sufficient condition for CP violation in the quark sector of the SM:

$$\Delta m^2_{tc}\Delta m^2_{tu}\Delta m^2_{cu}\Delta m^2_{bs}\Delta m^2_{bd}\Delta m^2_{sd}J_{\rm CKM} \neq 0. \tag{2.42}$$

Here

$$\Delta m^2_{ij} \equiv m_i^2 - m_j^2. \tag{2.43}$$

Equation (2.42) puts the following requirements on the SM in order that it violates CP:

(i) Within each quark sector, there should be no mass degeneracy;

(ii) None of the three mixing angles should be zero or $\pi/2$;

(iii) The phase should be neither 0 nor π.

As concerns the lepton sector of the ESM, the flavor parameters are the six lepton masses, and six mixing parameters: three mixing angles and three phases. One can parameterize $V_{\rm MNS}$ in a convenient way by factorizing it into $V_{\rm MNS}=VP$. Here P is a diagonal unitary matrix that depends on two phases, *e.g.* $P={\rm diag}(e^{i\phi_1},e^{i\phi_2},1)$, while V can be parametrized in the same way as (2.38). The advantage of this parametrization is that for the purpose of analyzing lepton number conserving processes and, in particular, neutrino flavor oscillations, the parameters of P are usually irrelevant and one can use the same Chau-Keung parametrization as is being used for $V_{\rm CKM}$. (An alternative way to understand these statements is to use a single-phase mixing matrix and put the extra two phases in the neutrino mass matrix. Then it is obvious that the effects of these 'Majorana-phases' always appear in conjunction with a factor of the Majorana mass that is lepton number violating parameter.) On the other hand, the Wolfenstein parametrization (2.39) is inappropriate for the lepton sector: it assumes $|V_{23}|\ll|V_{12}|\ll 1$, which does not hold here.

In order that the CP violating phase δ in V would be physically meaningful, *i.e.* there would be CP violation that is not related to lepton number violation, a condition similar to (2.42) should hold:

$$\Delta m^2_{\tau\mu}\Delta m^2_{\tau e}\Delta m^2_{\mu e}\Delta m^2_{32}\Delta m^2_{31}\Delta m^2_{21}J_{\rm MNS}\neq 0. \tag{2.44}$$

E. The Unitarity Triangles

A very useful concept is that of the *unitarity triangles*. We will focus on the quark sector, but analogous triangles can be defined in the lepton sector. The unitarity of the CKM matrix leads to various relations among the matrix elements, *e.g.*

$$V_{ud}V^*_{us}+V_{cd}V^*_{cs}+V_{td}V^*_{ts}=0, \tag{2.45}$$
$$V_{us}V^*_{ub}+V_{cs}V^*_{cb}+V_{ts}V^*_{tb}=0, \tag{2.46}$$
$$V_{ud}V^*_{ub}+V_{cd}V^*_{cb}+V_{td}V^*_{tb}=0. \tag{2.47}$$

Each of these three relations requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are "the unitarity triangles", though the term "unitarity triangle" is usually reserved for the relation (2.47) only. It is a surprising feature of the CKM matrix that all unitarity triangles are equal in area: the area of each unitarity triangle equals $|J_{\rm CKM}|/2$ while the sign of $J_{\rm CKM}$ gives the direction of the complex vectors around the triangles.

The rescaled unitarity triangle is derived from (2.47) by (a) choosing a phase convention such that $(V_{cd}V^*_{cb})$ is real, and (b) dividing the lengths of all sides by $|V_{cd}V^*_{cb}|$. Step (a) aligns one side of the triangle with the real axis, and step (b) makes the length of this side 1. The form of the triangle is unchanged. Two vertices of the rescaled unitarity triangle are thus fixed at (0,0) and (1,0). The coordinates of the remaining vertex correspond to the Wolfenstein parameters (ρ,η). The area of the rescaled unitarity triangle is $|\eta|/2$.

Depicting the rescaled unitarity triangle in the (ρ,η) plane, the lengths of the two complex sides are

$$R_u \equiv \left|\frac{V_{ud}V_{ub}}{V_{cd}V_{cb}}\right| = \sqrt{\rho^2+\eta^2}, \quad R_t \equiv \left|\frac{V_{td}V_{tb}}{V_{cd}V_{cb}}\right| = \sqrt{(1-\rho)^2+\eta^2}. \tag{2.48}$$

The three angles of the unitarity triangle are defined as follows [39,40]:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{2.49}$$

They are physical quantities and can be independently measured by CP asymmetries in B decays [41–45]. It is also useful to define the two small angles of the unitarity triangles (2.46) and (2.45):

$$\beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right], \quad \beta_K \equiv \arg\left[-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right]. \tag{2.50}$$

To make predictions for future measurements of CP violating observables, we need to find the allowed ranges for the CKM phases. There are three ways to determine the CKM parameters (see *e.g.* [46]):

(i) **Direct measurements** are related to SM tree level processes. At present, we have direct measurements of $|V_{ud}|$, $|V_{us}|$, $|V_{ub}|$, $|V_{cd}|$, $|V_{cs}|$, $|V_{cb}|$ and $|V_{tb}|$.

(ii) **CKM Unitarity** ($V_{\rm CKM}^\dagger V_{\rm CKM} = \mathbf{1}$) relates the various matrix elements. At present, these relations are useful to constrain $|V_{td}|$, $|V_{ts}|$, $|V_{tb}|$ and $|V_{cs}|$.

(iii) **Indirect measurements** are related to SM loop processes. At present, we constrain in this way $|V_{tb}V_{td}|$ (from Δm_B and Δm_{B_s}) and $\delta_{\rm KM}$ or, equivalently, η or β (from ε_K and $a_{\psi K_S}$).

When all available data are taken into account, one finds [47]:

$$\lambda = 0.2221 \pm 0.0021, \quad A = 0.827 \pm 0.058, \tag{2.51}$$

$$\rho = 0.23 \pm 0.11, \quad \eta = 0.37 \pm 0.08, \tag{2.52}$$

$$\sin 2\beta = 0.77 \pm 0.08, \quad \sin 2\alpha = -0.21 \pm 0.56, \quad 0.43 \lesssim \sin^2\gamma \leq 0.91. \tag{2.53}$$

Of course, there are correlations between the various parameters. The full information in the (ρ,η) plane is given in fig. 1 [47].

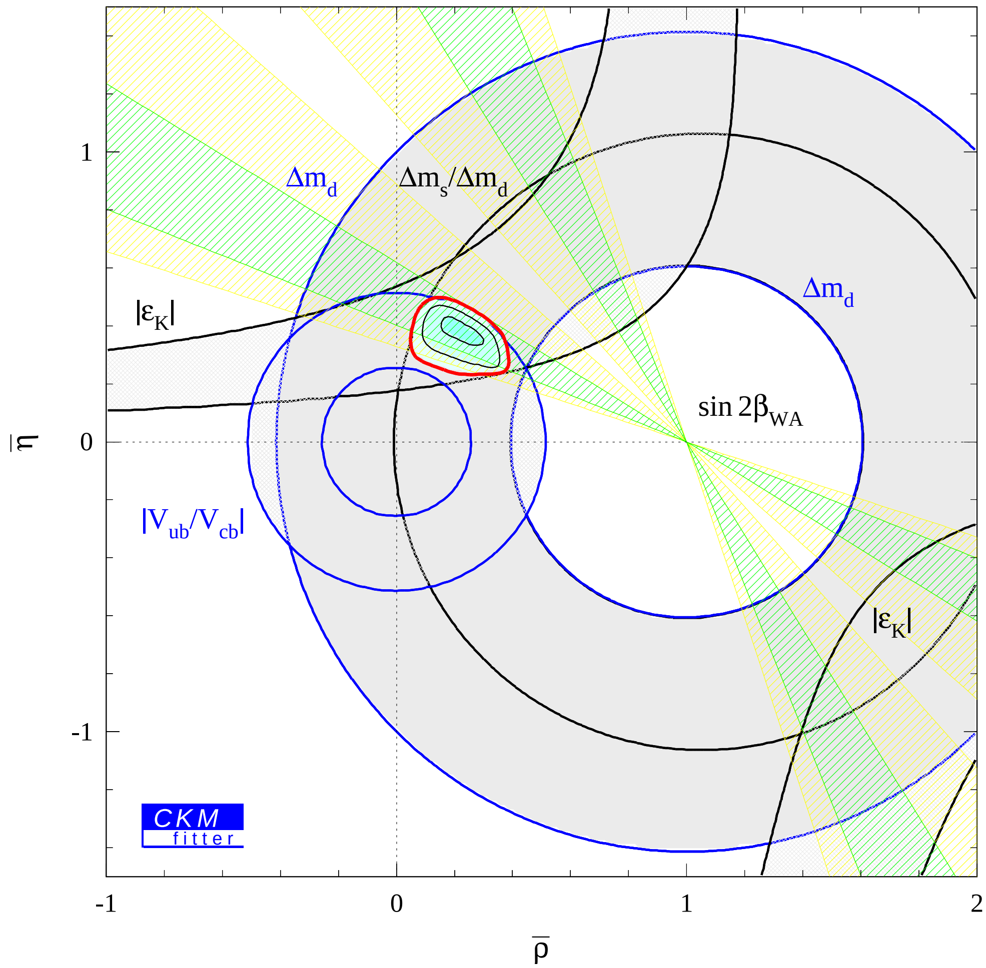

FIG. 1. Present Standard Model constraints and the result from the global CKM fit.

F. The Uniqueness of the Standard Model Picture of CP Violation

In the previous subsections, we have learnt several features of CP violation as explained by the Standard Model. It is important to understand that various reasonable (and often well-motivated) extensions of the SM provide examples where some or all of these features do not hold. Furthermore, until a few years ago, none of the special features of the Kobayashi-Maskawa mechanism of CP violation has been experimentally tested. This situation has dramatically changed recently. Let us survey some of the SM features, how they can be modified with new physics, and whether experiment has shed light on these questions.

(i) $\delta_{\rm KM}$ *is the only source of CP violation in meson decays.* This is arguably the most unique feature of the SM and gives the model a strong predicitive power. It is violated in almost any low-energy extension. For example, in the supersymmetric extension of the SM there are 44 physical CP violating phases, many of which affect meson decays. The measured value of $a_{\psi K_S}$ is consistent with the correlation between K and B decays that is predicted by the SM. It is therefore very likely that $\delta_{\rm KM}$ is indeed the dominant source of CP violation in meson decays.

(ii) *CP violation is small in $K \to \pi\pi$ decays because of flavor suppression and not because CP is an approximate symmetry.* In many (though certainly not all) supersymmetric models, the flavor suppression is too mild, or entirely ineffective, requiring approximate CP to hold. The measurement of $a_{\psi K_S} = \mathcal{O}(1)$ confirms that not all CP violating phases are small.

(iii) *CP violation appears in both $\Delta F = 1$ (decay) and $\Delta F = 2$ (mixing) amplitudes.* Superweak models suggest that CP is violated only in mixing amplitudes. The measurement of ε'/ε confirms that there is CP violation in $\Delta S = 1$ processes.

(iv) *CP is not violated in the lepton sector.* Models that allow for neutrino masses, such as the ESM framework presented above, predict CP violation in leptonic charged current interactions. The data from neutrino oscillation experiments makes it very likely that charged current weak interactions violate CP also in the lepton sector.

(v) *CP violation appears only in the charged current weak interactions and in conjunction with flavor changing processes.* Here both various extensions of the SM (such as supersymmetry) and non-perturbative effects within the SM ($\theta_{\rm QCD}$) allow for CP violation in other types of interactions and in flavor diagonal processes. In particular, it is difficult to avoid flavor-diagonal phases in the supersymmetric framework. The fact that no electric dipole moment has been measured yet poses difficulties to many models with diagonal CP violation (and, of course, is responsible to the strong CP problem within the SM).

(vi) *CP is explicitly broken.* In various extensions of the scalar sector, it is possible to achieve spontaneous CP violation. It will be very difficult to test this question experimentally.

This situation, where the Standard Model has a very unique and predictive description of CP violation and the number of experimentally measured CP violating observables is very limited (ε_K, ε'/ε and $a_{\psi K_S}$), is the basis for the strong interest, experimental and theoretical, in CP violation. There are two types of unambiguous tests concerning CP violation in the Standard Model: First, since there is a single source of CP violation, all observables are correlated with each other. For example, the CP asymmetries in $B \to \psi K_S$ and in $K \to \pi\nu\bar{\nu}$ are strongly correlated [48–50]. Second, since CP violation is restricted to flavor changing fermion processes, it is predicted to be highly suppressed in the lepton sector and practically vanish in flavor diagonal processes. For example, the transverse lepton polarization in semileptonic meson decays, CP violation in $t\bar{t}$ production, and (assuming $\theta_{\rm QCD} = 0$) the electric dipole moment of the neutron are all predicted to be orders of magnitude below the (present and near future) experimental sensitivity. We conclude that it is highly important to search for CP violation in many different systems.

III. MESON DECAYS

In the previous section, we explained how CP violation arises in the Standard Model. In the next three sections, we would like to understand the implications of this theory for the phenomenology of CP violation in K, D and B decays. To do so, we first present a model independent analysis of CP violation in meson decays.

We distinguish between three different types of CP violation in meson decays:

(i) CP violation in mixing, which occurs when the two neutral mass eigenstate admixtures cannot be chosen to be CP-eigenstates;

(ii) CP violation in decay, which occurs in both charged and neutral decays, when the amplitude for a decay and its CP-conjugate process have different magnitudes;

(iii) CP violation in the interference of decays with and without mixing, which occurs in decays into final states that are common to B^0 and $\bar{B}^0$.

A. Notations and Formalism

To define these three types and to discuss their theoretical calculation and experimental measurement, we first introduce some notations and formalism. We refer specifically to B meson mixing and decays, but most of our discussion applies equally well to K, B_s and D mesons.

A B^0 meson is made from a b-type antiquark and an d-type quark, while the $\bar{B}^0$ meson is made from a b-type quark and an d-type antiquark. Our phase convention for the CP transformation law of the neutral B mesons is defined by

$$\mathrm{CP}|B^0\rangle = \omega_B|\bar{B}^0\rangle, \quad \mathrm{CP}|\bar{B}^0\rangle = \omega_B^*|B^0\rangle, \quad (|\omega_B| = 1). \tag{3.1}$$

Physical observables do not depend on the phase factor ω_B.

The light, B_L, and heavy, B_H, mass eigenstates can be written as linear combinations of B^0 and $\bar{B}^0$:

$$\begin{aligned} |B_L\rangle &= p|B^0\rangle + q|\bar{B}^0\rangle, \\ |B_H\rangle &= p|B^0\rangle - q|\bar{B}^0\rangle, \end{aligned} \tag{3.2}$$

with

$$|q|^2 + |p|^2 = 1. \tag{3.3}$$

The mass difference Δm_B and the width difference $\Delta\Gamma_B$ are defined as follows:

$$\Delta m \equiv M_H - M_L, \quad \Delta\Gamma \equiv \Gamma_H - \Gamma_L. \tag{3.4}$$

The average mass and width are given by

$$m_B \equiv \frac{M_H + M_L}{2}, \quad \Gamma_B \equiv \frac{\Gamma_H + \Gamma_L}{2}. \tag{3.5}$$

It is useful to define dimensionless ratios x and y:

$$x \equiv \frac{\Delta m}{\Gamma}, \quad y \equiv \frac{\Delta\Gamma}{2\Gamma}. \tag{3.6}$$

The time evolution of the mass eigenstates is simple:

$$\begin{aligned} |B_H(t)\rangle &= e^{-iM_H t} e^{-\Gamma_H t/2}|B_H\rangle, \\ |B_L(t)\rangle &= e^{-iM_L t} e^{-\Gamma_L t/2}|B_L\rangle. \end{aligned} \tag{3.7}$$

The time evolution of the strong interaction eigenstates is complicated and obeys a Schrödinger-like equation,

$$i\frac{d}{dt}\begin{pmatrix} B \\ \bar{B} \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} B \\ \bar{B} \end{pmatrix}, \tag{3.8}$$

where M and Γ are 2×2 Hermitian matrices.

The off-diagonal terms in these matrices, M_{12} and Γ_{12}, are particularly important in the discussion of mixing and CP violation. M_{12} is the dispersive part of the transition amplitude from B^0 to $\bar{B}^0$, while Γ_{12} is the absorptive part of that amplitude.

Solving the eigenvalue equation gives

$$(\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2 = (4|M_{12}|^2 - |\Gamma_{12}|^2), \qquad \Delta m \Delta\Gamma = 4\mathrm{Re}(M_{12}\Gamma_{12}^*), \tag{3.9}$$

$$\frac{q}{p} = -\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - \frac{i}{2}\Delta\Gamma} = -\frac{\Delta m - \frac{i}{2}\Delta\Gamma}{2M_{12} - i\Gamma_{12}}. \tag{3.10}$$

In the B system, $|\Gamma_{12}| \ll |M_{12}|$ (see discussion below), and then, to leading order in $|\Gamma_{12}/M_{12}|$, eqs. (3.9) and (3.10) can be written as

$$\Delta m_B = 2|M_{12}|, \quad \Delta\Gamma_B = 2\mathrm{Re}(M_{12}\Gamma_{12}^*)/|M_{12}|, \tag{3.11}$$

$$q/p = -M_{12}^*/|M_{12}|. \tag{3.12}$$

To discuss CP violation in mixing, it is useful to write eq. (3.10) to first order in $|\Gamma_{12}/M_{12}|$ [rather than to zeroth order as in (3.12)]:

$$\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|}\left[1 - \frac{1}{2}\mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)\right]. \tag{3.13}$$

To discuss CP violation in decay, we need to consider decay amplitudes. The CP transformation law for a final state f is

$$\mathrm{CP}|f\rangle = \omega_f|\bar{f}\rangle, \quad \mathrm{CP}|\bar{f}\rangle = \omega_f^*|f\rangle, \quad (|\omega_f| = 1). \tag{3.14}$$

For a final CP eigenstate $f = \bar{f} = f_{\mathrm{CP}}$, the phase factor ω_f is replaced by $\eta_{f_{\mathrm{CP}}} = \pm 1$, the CP eigenvalue of the final state. We define the decay amplitudes A_f and $\bar{A}_f$ according to

$$A_f = \langle f|\mathcal{H}_d|B^0\rangle, \quad \bar{A}_f = \langle f|\mathcal{H}_d|\bar{B}^0\rangle, \tag{3.15}$$

where $\mathcal{H}_d$ is the decay Hamiltonian.

CP relates A_f and $\bar{A}_{\bar{f}}$. There are two types of phases that may appear in A_f and $\bar{A}_{\bar{f}}$. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the CP-conjugate amplitude. Thus their phases appear in A_f and $\bar{A}_{\bar{f}}$ with opposite signs. In the SM these phases occur only in the mixing matrices that parameterize the charged current weak interactions, hence these are often called "weak phases". The weak phase of any single term is convention dependent. However the difference between the weak phases in two different terms in A_f is convention independent because the phase rotations of the initial and final states are the same for every term. A second type of phase can appear in scattering or decay amplitudes even when the Lagrangian is real. Such phases do not violate CP and they appear in A_f and $\bar{A}_{\bar{f}}$ with the same sign. Their origin is the possible contribution from intermediate on-shell states in the decay process, that is an absorptive part of an amplitude that has contributions from coupled channels. Usually the dominant rescattering is due to strong interactions and hence the designation "strong phases" for the phase shifts so induced. Again only the relative strong phases of different terms in a scattering amplitude have physical content, an overall phase rotation of the entire amplitude has no physical consequences. Thus it is useful to write each contribution to A in three parts: its magnitude A_i; its weak phase term $e^{i\phi_i}$; and its strong phase term $e^{i\delta_i}$. Then, if several amplitudes contribute to $B \to f$, we have

$$\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| = \left|\frac{\sum_i A_i e^{i(\delta_i - \phi_i)}}{\sum_i A_i e^{i(\delta_i + \phi_i)}}\right|. \tag{3.16}$$

To discuss CP violation in the interference of decays with and without mixing, we introduce a complex quantity λ_f defined by

$$\lambda_f \; = \; \frac{q}{p}\,\frac{\bar{A}_f}{A_f}. \tag{3.17}$$

We further define the CP transformation law for the quark fields in the Hamiltonian (a careful treatment of CP conventions can be found in [51]):

$$q \;\to\; \omega_q \bar{q}, \quad \bar{q} \;\to\; \omega_q^* q, \quad (|\omega_q| = 1). \tag{3.18}$$

The effective Hamiltonian that is relevant to M_{12} is of the form

$$H_{\rm eff}^{\Delta b=2} \propto e^{+2i\phi_B}\left[\bar{d}\gamma^\mu(1-\gamma_5)b\right]^2 + e^{-2i\phi_B}\left[\bar{b}\gamma^\mu(1-\gamma_5)d\right]^2, \tag{3.19}$$

where $2\phi_B$ is a CP violating (weak) phase. (We use the SM $V-A$ amplitude, but the results can be generalized to any Dirac structure.) For the B system, where $|\Gamma_{12}| \ll |M_{12}|$, this leads to

$$q/p = \omega_B \omega_b^* \omega_d e^{-2i\phi_B}. \tag{3.20}$$

(We implicitly assumed that the vacuum insertion approximation gives the correct sign for M_{12}. In general, there is a sign(B_B) factor on the right hand side of eq. (3.20) [52].) To understand the phase structure of decay amplitudes, we take as an example the $b \to q\bar{q}d$ decay ($q = u$ or c). The decay Hamiltonian is of the form

$$H_d \propto e^{+i\phi_f}\left[\bar{q}\gamma^\mu(1-\gamma_5)d\right]\left[\bar{b}\gamma_\mu(1-\gamma_5)q\right] + e^{-i\phi_f}\left[\bar{q}\gamma^\mu(1-\gamma_5)b\right]\left[\bar{d}\gamma_\mu(1-\gamma_5)q\right], \tag{3.21}$$

where ϕ_f is the appropriate weak phase. (Again, for simplicity we use a $V-A$ structure, but the results hold for any Dirac structure.) Then

$$\bar{A}_{\bar{f}}/A_f = \omega_f \omega_B^* \omega_b \omega_d^* e^{-2i\phi_f}. \tag{3.22}$$

Eqs. (3.20) and (3.22) together imply that for a final CP eigenstate,

$$\lambda_{f_{\rm CP}} = \eta_{f_{\rm CP}} e^{-2i(\phi_B + \phi_f)}. \tag{3.23}$$

B. The Three Types of CP Violation in Meson Decays

1. CP violation in mixing

$$|q/p| \neq 1. \tag{3.24}$$

This type of CP violation results from the mass eigenstates being different from the CP eigenstates, and requires a relative phase between M_{12} and Γ_{12}. For the neutral B system, this effect could be observed through the asymmetries in semileptonic decays:

$$a_{\rm SL} = \frac{\Gamma(\bar{B}^0_{\rm phys}(t) \to \ell^+ \nu X) - \Gamma(B^0_{\rm phys}(t) \to \ell^- \nu X)}{\Gamma(\bar{B}^0_{\rm phys}(t) \to \ell^+ \nu X) + \Gamma(B^0_{\rm phys}(t) \to \ell^- \nu X)}. \tag{3.25}$$

In terms of q and p,

$$a_{\rm SL} = \frac{1 - |q/p|^4}{1 + |q/p|^4}. \tag{3.26}$$

CP violation in mixing has been observed in the neutral K system (Re $\varepsilon_K \neq 0$).

In the neutral B system, the effect is expected to be small, $\lesssim \mathcal{O}(10^{-2})$. The reason is that, model independently, the effect cannot be larger than $\mathcal{O}(\Delta\Gamma_B/\Delta m_B)$. The difference in width is produced by decay channels common to B^0 and $\bar{B}^0$. The branching ratios for such channels are at or below the level of 10^{-3}. Since various channels contribute with differing signs, one expects that their sum does not exceed the individual level. Hence, we can safely assume that $\Delta\Gamma_B/\Gamma_B = \mathcal{O}(10^{-2})$. On the other hand, it is experimentaly known that $\Delta m_B/\Gamma_B \approx 0.7$.

To calculate $a_{\rm SL}$, we use (3.26) and (3.13), and get:

$$a_{\rm SL} = {\rm Im}(\Gamma_{12}/M_{12}). \tag{3.27}$$

To predict it in a given model, one needs to calculate M_{12} and Γ_{12}. This involves large hadronic uncertainties, in particular in the hadronization models for Γ_{12}.

2. CP violation in decay

$$|\bar{A}_{\bar{f}}/A_f| \neq 1. \tag{3.28}$$

This appears as a result of interference among various terms in the decay amplitude, and will not occur unless at least two terms have different weak phases and different strong phases. CP asymmetries in charged B decays,

$$a_{f^\pm} = \frac{\Gamma(B^+ \to f^+) - \Gamma(B^- \to f^-)}{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)}, \tag{3.29}$$

are purely an effect of CP violation in decay. In terms of the decay amplitudes,

$$a_{f^\pm} = \frac{1 - |\bar{A}_{f^-}/A_{f^+}|^2}{1 + |\bar{A}_{f^-}/A_{f^+}|^2}. \tag{3.30}$$

CP violation in decay has been observed in the neutral K system (Re $\varepsilon'_K \neq 0$).

To calculate $a_{f^\pm}$, we use (3.30) and (3.16). For simplicity, we consider decays with contributions from two weak phases and with $A_2 \ll A_1$. We get:

$$a_{f^\pm} = -2(A_2/A_1)\sin(\delta_2 - \delta_1)\sin(\phi_2 - \phi_1). \tag{3.31}$$

The magnitude and strong phase of any amplitude involve long distance strong interaction physics, and our ability to calculate these from first principles is limited. Thus quantities that depend only on the weak phases are much cleaner than those that require knowledge of the relative magnitudes or strong phases of various amplitude contributions, such as CP violation in decay.

3. CP violation in the interference between decays with and without mixing

$$\mathrm{Im}\ \lambda_{f_{\rm CP}} \neq 0. \tag{3.32}$$

This effect is the result of interference between a direct decay amplitude and a first-mix-then-decay path to the same final state. For the neutral B system, the effect can be observed by comparing decays into final CP eigenstates of a time-evolving neutral B state that begins at time zero as B^0 to those of the state that begins as $\bar{B}^0$:

$$a_{f_{\rm CP}}(t) = \frac{\Gamma(\bar{B}^0_{\rm phys}(t) \to f_{\rm CP}) - \Gamma(B^0_{\rm phys}(t) \to f_{\rm CP})}{\Gamma(\bar{B}^0_{\rm phys}(t) \to f_{\rm CP}) + \Gamma(B^0_{\rm phys}(t) \to f_{\rm CP})}. \tag{3.33}$$

This time dependent asymmetry is given, in general, by

$$a_{f_{\rm CP}}(t) = -\frac{1-|\lambda_{f_{\rm CP}}|^2}{1+|\lambda_{f_{\rm CP}}|^2}\cos(\Delta m_B t) + \frac{2{\rm Im}\lambda_{f_{\rm CP}}}{1+|\lambda_{f_{\rm CP}}|^2}\sin(\Delta m_B t). \tag{3.34}$$

In decays with $|\lambda_{f_{\rm CP}}| = 1$, (3.32) is the only contributing effect:

$$a_{f_{\rm CP}}(t) = {\rm Im}\lambda_{f_{\rm CP}}\sin(\Delta m_B t). \tag{3.35}$$

We often use

$$a_{f_{\rm CP}} \equiv \frac{2{\rm Im}\lambda_{f_{\rm CP}}}{1+|\lambda_{f_{\rm CP}}|^2}. \tag{3.36}$$

CP violation in the interference of decays with and without mixing has been observed for the neutral K system (Im $\varepsilon_K \neq 0$) and for the neutral B system ($a_{\psi K_S} \neq 0$). In the latter, it is an effect of $\mathcal{O}(1)$. For such cases, the contribution from CP violation in mixing is clearly negligible. For decays that are dominated by a single CP violating phase (for example, $B \to \psi K_S$ and $K_L \to \pi^0\nu\bar{\nu}$), so that the contribution from CP violation in decay is also negligible, $a_{f_{\rm CP}}$ is cleanly interpreted in terms of purely electroweak parameters. Explicitly, ${\rm Im}\lambda_{f_{\rm CP}}$ gives the relative phase between the $B-\bar{B}$ mixing amplitude and the relevant decay amplitudes [see eq. (3.23)]:

$${\rm Im}\lambda_{f_{\rm CP}} = -\eta_{f_{\rm CP}}\sin[2(\phi_B + \phi_f)]. \tag{3.37}$$

4. Direct and Indirect CP Violation

The terms indirect CP violation and direct CP violation are commonly used in the literature. While various authors use these terms with different meanings, the most useful definition is the following:

Indirect CP violation refers to CP violation in meson decays where the CP violating phases can all be chosen to appear in $\Delta F = 2$ (mixing) amplitudes.

Direct CP violation refers to CP violation in meson decays where some CP violating phases necessarily appear in $\Delta F = 1$ (decay) amplitudes.

Examining eqs. (3.24) and (3.10), we learn that CP violation in mixing is a manifestation of indirect CP violation. Examining eqs. (3.28) and (3.15), we learn that CP violation in decay is a manifestation of direct CP violation. Examining eqs. (3.32) and (3.17), we learn that the situation concerning CP violation in the interference of decays with and without mixing is more subtle. For any single measurement of $\mathrm{Im}\lambda_f \neq 0$, the relevant CP violating phase can be chosen by convention to reside in the $\Delta F = 2$ amplitude [$\phi_f = 0$, $\phi_B \neq 0$ in the notation of eq. (3.23)], and then we would call it indirect CP violation. Consider, however, the CP asymmetries for two different final CP eigenstates (for the same decaying meson), f_a and f_b. Then, a non-zero difference between $\mathrm{Im}\lambda_{f_a}$ and $\mathrm{Im}\lambda_{f_b}$ requires that there exists CP violation in $\Delta F = 1$ processes ($\phi_{f_a} - \phi_{f_b} \neq 0$), namely direct CP violation.

Experimentally, both direct and indirect CP violation have been established. Below we will see that ε_K signifies indirect CP violation while ε'_K signifies direct CP violation.

Theoretically, most models of CP violation (including the Standard Model) have predicted that both types of CP violation exist. There is, however, one class of models, that is *superweak models*, that predict only indirect CP violation. The measurement of $\varepsilon'_K \neq 0$ has excluded this class of models.

IV. K DECAYS

Measurements of CP violation have played an enormous role in particle physics. First, the measurement of ε_K in 1964 provided the first evidence that CP is not a symmetry of Nature. This discovery revolutionized the thinking of particle physicists and was essential for understanding baryogenesis. Second, the measurement in 1988 of ε'_K provided the first evidence for direct CP violation and excluded the superweak scenario. In the future, the search for CP violation in $K \to \pi\nu\bar{\nu}$ decays will add significantly to our understanding of CP violation.

A. ε_K and ε'_K

Historically, a different language from the one used by us has been employed to describe CP violation in $K \to \pi\pi$ and $K \to \pi\ell\nu$ decays. In this section we ‘translate’ the language of ε_K and ε'_K to our notations. Doing so will make it easy to understand which type of CP violation is related to each quantity.

The two CP violating quantities measured in neutral K decays are

$$\eta_{00} = \frac{\langle \pi^0\pi^0|\mathcal{H}|K_L\rangle}{\langle \pi^0\pi^0|\mathcal{H}|K_S\rangle}, \quad \eta_{+-} = \frac{\langle \pi^+\pi^-|\mathcal{H}|K_L\rangle}{\langle \pi^+\pi^-|\mathcal{H}|K_S\rangle}. \tag{4.1}$$

Define, for $(ij) = (00)$ or $(+-)$,

$$A_{ij} = \langle \pi^i\pi^j|\mathcal{H}|K^0\rangle, \quad \bar{A}_{ij} = \langle \pi^i\pi^j|\mathcal{H}|\bar{K}^0\rangle, \quad \lambda_{ij} = \left(\frac{q}{p}\right)_K \frac{\bar{A}_{ij}}{A_{ij}}. \tag{4.2}$$

Then

$$\eta_{00} = \frac{1-\lambda_{00}}{1+\lambda_{00}}, \quad \eta_{+-} = \frac{1-\lambda_{+-}}{1+\lambda_{+-}}. \tag{4.3}$$

The η_{00} and η_{+-} parameters get contributions from CP violation in mixing ($|(q/p)|_K \neq 1$) and from the interference of decays with and without mixing ($\mathrm{Im}\lambda_{ij} \neq 0$) at $\mathcal{O}(10^{-3})$ and from CP violation in decay ($|\bar{A}_{ij}/A_{ij}| \neq 1$) at $\mathcal{O}(10^{-6})$.

There are two isospin channels in $K \to \pi\pi$ leading to final $(2\pi)_{I=0}$ and $(2\pi)_{I=2}$ states:

$$\begin{aligned} \langle \pi^0\pi^0| = &\ \sqrt{1/3}\langle(\pi\pi)_{I=0}| - \sqrt{2/3}\langle(\pi\pi)_{I=2}|, \\ \langle \pi^+\pi^-| = &\ \sqrt{2/3}\langle(\pi\pi)_{I=0}| + \sqrt{1/3}\langle(\pi\pi)_{I=2}|. \end{aligned} \tag{4.4}$$

The fact that there are two strong phases allows for CP violation in decay. The possible effects are, however, small (on top of the smallness of the relevant CP violating phases) because the final $I = 0$ state is dominant (this is the $\Delta I = 1/2$ rule). Define

$$A_I = \langle (\pi\pi)_I|\mathcal{H}|K^0\rangle, \quad \bar{A}_I = \langle (\pi\pi)_I|\mathcal{H}|\bar{K}^0\rangle, \quad \lambda_I = \left(\frac{q}{p}\right)_K \left(\frac{\bar{A}_I}{A_I}\right). \tag{4.5}$$

Experimentally, $|A_2/A_0| \approx 1/20$. Instead of η_{00} and η_{+-} we may define two combinations, ε_K and ε'_K, in such a way that the possible effects of direct (indirect) CP violation are isolated into ε'_K (ε_K).

The experimental definition of the ε_K parameter is

$$\varepsilon_K \equiv \frac{1}{3}(\eta_{00} + 2\eta_{+-}). \tag{4.6}$$

The experimental value is given by eq. (1.1). To zeroth order in A_2/A_0, we have $\eta_{00} = \eta_{+-} = \varepsilon_K$. However, the specific combination (4.6) is chosen in such a way that the following relation holds to *first* order in A_2/A_0:

$$\varepsilon_K = \frac{1-\lambda_0}{1+\lambda_0}. \tag{4.7}$$

Since, by definition, only one strong channel contributes to λ_0, there is indeed no CP violation in decay in (4.7). It is simple to show that Re $\varepsilon_K \neq 0$ is a manifestation of CP violation in mixing while Im $\varepsilon_K \neq 0$ is a manifestation of CP violation in the interference between decays with and without mixing. Since experimentally $\arg\varepsilon_K \approx \pi/4$, the two contributions are comparable. It is also clear that $\varepsilon_K \neq 0$ is a manifestation of indirect CP violation: it could be described entirely in terms of a CP violating phase in the M_{12} amplitude.

The experimental definition of the ε'_K parameter is

$$\varepsilon'_K \equiv \frac{1}{3}(\eta_{+-} - \eta_{00}). \tag{4.8}$$

The quantity that is actually measured in experiment is

$$1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = 6\mathrm{Re}(\varepsilon'/\varepsilon). \tag{4.9}$$

The world average is given in eq. (1.2). The theoretical expression is

$$\varepsilon'_K \approx \frac{1}{6}(\lambda_{00} - \lambda_{+-}). \tag{4.10}$$

Obviously, any type of CP violation which is independent of the final state does not contribute to ε'_K. Consequently, there is no contribution from CP violation in mixing to (4.10). It is simple to show that Re $\varepsilon'_K \neq 0$ is a manifestation of CP violation in decay while Im $\varepsilon'_K \neq 0$ is a manifestation of CP violation in the interference between decays with and without mixing. Following our explanations in the previous section, we learn that $\varepsilon'_K \neq 0$ is a manifestation of direct CP violation: it requires $\phi_2 - \phi_0 \neq 0$ [where ϕ_I is the CP violating phase in the A_I amplitude defined in (4.5)].

1. The ε_K Parameter in the Standard Model

An approximate expression for ε_K, that is convenient for calculating it, is given by

$$\varepsilon_K = \frac{e^{i\pi/4}}{\sqrt{2}} \frac{\mathrm{Im}M_{12}}{\Delta m_K}. \tag{4.11}$$

A few points concerning this expression are worth emphasizing:

(i) Eq. (4.11) is given in a specific phase convention, where A_2 is real. Within the SM, this is a phase convention where $V_{ud}V^*_{us}$ is real, a condition fulfilled in both the standard parametrization of eq. (2.38) and the Wolfenstein parametrization of eq. (2.39).

(ii) The phase of $\pi/4$ is approximate. It is determined by hadronic parameters and therefore is independent of the electroweak model. Specifically,

$$\arg(\varepsilon_K) \approx \arctan(-2\Delta m_K/\Delta\Gamma_K) \approx \pi/4. \tag{4.12}$$

(iii) A term of order $2\frac{\mathrm{Im}\ A_0}{\mathrm{Re}\ A_0}\frac{\mathrm{Re}\ M_{12}}{\mathrm{Im}\ M_{12}} \lesssim 0.02$ is neglected when (4.11) is derived.

(iv) There is a large hadronic uncertainty in the calculation of M_{12} coming from long distance contributions. There are, however, good reasons to believe that the long distance contributions are important in Re M_{12} (where they could be even comparable to the short distance contributions), but negligible in Im M_{12}. To avoid this uncertainty, one uses

$\mathrm{Im}M_{12}/\Delta m_K$ with the experimentally measured value of Δm_K, instead of $\mathrm{Im}M_{12}/2\mathrm{Re}\ M_{12}$ with the theoretically calculated value of Re M_{12}.

(v) The matrix element $\langle \bar{K}^0|(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}|K^0\rangle$ is yet another source of hadronic uncertainty. If both Im M_{12} and Re M_{12} were dominated by short distance contributions, one would use the ratio $\mathrm{Im}M_{12}/\mathrm{Re}\ M_{12}$ where the matrix element cancels out. However, as explained above, this is not the case.

Within the Standard Model, Im M_{12} is accounted for by box diagrams. We follow here the notations of ref. [53], where precise definitions, numerical values and appropriate references are given. One obtains:

$$\varepsilon_K = e^{i\pi/4} C_\varepsilon B_K \mathrm{Im}(V_{ts}^* V_{td}) \left\{ \mathrm{Re}(V_{cs}^* V_{cd})[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)] - \mathrm{Re}(V_{ts}^* V_{td}) \eta_2 S_0(x_t) \right\}, \tag{4.13}$$

where $C_\varepsilon \equiv \frac{G_F^2 f_K^2 m_K m_W^2}{6\sqrt{2}\pi^2 \Delta m_K}$ is a well known parameter, the η_i are QCD correction factors, S_0 is a kinematic factor, and B_K is the ratio between the matrix element of the four quark operator and its value in the vacuum insertion approximation.

We would like to emphasize the following points:

(i) CP violation was discovered through the measurement of ε_K. Hence this measurement played a significant role in the history of particle physics.

(ii) For a long time, ε_K has been the only measured CP violating parameter. Roughly speaking, this measurement set the value of δ_{KM} (and, by requiring $\delta_{\mathrm{KM}} = \mathcal{O}(1)$, made the KM mechanism plausible) but could not serve as a test of the KM mechanism. (More precisely, a value of $|\varepsilon_K| \gg 10^{-3}$ would have invalidated the KM mechanism, but any value $|\varepsilon_K| \lesssim 10^{-3}$ was acceptable.) It is only the combination of the new measurement of $a_{\psi K_S}$ with ε_K that provides the first precision test of the KM mechanism.

(iii) Within the SM, the smallness of ε_K is not related to suppression of CP violation but rather to suppression of flavor violation. Specifically, it is the smallness of the ratio $|(V_{td}V_{ts})/(V_{ud}V_{us})| \sim \lambda^4$ that explains $|\varepsilon_K| \sim 10^{-3}$.

(iv) Until recently, the measured value of ε_K provided a unique type of information on the CKM phase. For example, the measurement of sign(Re ε_K) > 0 tells us that $\eta > 0$ and excludes the lower half of the $\rho - \eta$ plane. Such information cannot be obtained from any CP conserving observable.

(v) The ε_K constraint gives hyperbolae in the $\rho - \eta$ plane. It is shown in fig. 1. The measured value is consistent with all other CKM-related measurements and further narrows the allowed region.

(vi) The main sources of uncertainty are in the B_K parameter, $B_K = 0.85 \pm 0.15$, and in the $|V_{cb}|^4$ dependence.

(vii) ε_K is an extremely powerful probe of new physics. Its small value poses a problem to any model of new physics where the flavor suppression is less efficient than the GIM mechanism [54] of the SM. For example, the construction of viable supersymmetric models is highly constrained by the requirement that they do not give contributions that are orders of magnitude higher than the experimental value.

Direct CP violation in $K \to \pi\pi$ decays was first measured in 1988 [3]. Two recent measurements achieved impressive accuracy:

$$\frac{\varepsilon'}{\varepsilon} = \begin{cases} (20.7 \pm 2.8) \times 10^{-4} & \text{KTeV [6]}, \\ (15.3 \pm 2.6) \times 10^{-4} & \text{NA48 [7]}. \end{cases} \tag{4.14}$$

In combination with previous results [4,5], the present world average has an accuracy of order 10% [see eq. (1.2)].

A convenient approximate expression for ε'_K is given by:

$$\varepsilon'_K = \frac{i}{\sqrt{2}} \left| \frac{A_2}{A_0} \right| e^{i(\delta_2 - \delta_0)} \sin(\phi_2 - \phi_0). \tag{4.15}$$

We would like to emphasize a few points:

(i) The approximations used in (4.15) are $|q/p| = 1$ and $|A_2/A_0| \ll 1$.

(ii) The phase of ε'_K is determined by hadronic parameters and is, therefore, model independent: $\arg(\varepsilon'_K) = \pi/2 + \delta_2 - \delta_0 \approx \pi/4$. The fact that, accidentally, $\arg(\varepsilon_K) \approx \arg(\varepsilon'_K)$, means that

$$\mathrm{Re}(\varepsilon'/\varepsilon) \approx \varepsilon'/\varepsilon. \tag{4.16}$$

(iii) Re $\varepsilon'_K \neq 0$ requires $\delta_2 - \delta_0 \neq 0$, consistent with our statement that it is a manifestation of CP violation in decay. $\varepsilon'_K \neq 0$ requires $\phi_2 - \phi_0 \neq 0$, consistent with our statement that it is a manifestation of direct CP violation.

The calculation of ε'/ε within the Standard Model suffers from large hadronic uncertainties. A very naive order of magnitude estimate gives $\varepsilon'/\varepsilon \sim (A_2/A_0)(A_0^{\rm penguin}/A_0^{\rm tree}) \sim 10^{-3}$. Note that ε'/ε is not small because of small CP violating parameters but because of hadronic parameters.

The value of the phase β_K cancels in the ratio ε'/ε and therefore did not affect our estimate. In actual calculations, one usually uses the experimental value of ε_K and the theoretical expression for ε'_K. Then the expression for ε'/ε depends on the CP violating phase.

The detailed calculation of ε'/ε is complicated. There are several comparable contributions with differing signs. The final result can be written in the form (for details and references, see [53]):

$$\begin{aligned} \varepsilon'/\varepsilon &= \mathrm{Im}(V_{td}V_{ts}^*) \left[P^{(1/2)} - P^{(3/2)} \right] \\ &\approx 13\ \mathrm{Im}(V_{td}V_{ts}^*) \left(\frac{110\ MeV}{m_s(2\ GeV)} \right)^2 \left(\frac{\Lambda^{(4)}_{\overline{\rm MS}}}{340\ MeV} \right) \\ &\times \left[B_6^{(1/2)}(1 - \Omega_{\eta+\eta'}) - 0.4 B_8^{(3/2)} \left(\frac{m_t}{165\ GeV} \right)^{2.5} \right]. \end{aligned} \tag{4.17}$$

We omitted here a phase factor using the approximation $\arg(\varepsilon_K) = \arg(\varepsilon'_K)$. Here $P^{(1/2)}$, which is dominated by QCD penguins, gives the contributions from $\Delta I = 1/2$ transitions, while $P^{(3/2)}$, which is dominated by electroweak penguins, gives the contributions from $\Delta I = 3/2$ transitions. The $B_6^{(1/2)}$ and $B_8^{(3/2)}$ factors parameterize the corresponding hadronic matrix elements. The QCD penguin contributions are suppressed by isospin breaking effects ($m_u \neq m_d$), parametrized by $\Omega_{\eta+\eta'}$. The resulting estimates vary in the range [53]:

$$\mathrm{Re}(\varepsilon'/\varepsilon)^{\mathrm{SM}} = (0.5 - 4) \times 10^{-3}. \tag{4.18}$$

We would like to emphasize the following points:

(i) Direct CP violation was discovered through the measurement of ε'.

(ii) The SM range (4.18) is consistent with the experimental result (1.2).

(iii) The main sources of uncertainties lie then in the parameters m_s, $B_6^{(1/2)}$, $B_8^{(3/2)}$, $\Omega_{\eta+\eta'}$ and $\Lambda^{(4)}_{\overline{\mathrm{MS}}}$. The importance of these uncertainties is increased because of the cancellation between the two contributions in (4.17).

(iv) The large hadronic uncertainties make it difficult to use the experimental value of ε'/ε to constrain the CKM parameters. Still, a negative value or a value much smaller than 10^{-4} would have been very puzzling in the context of the SM.

(v) The experimental result is useful in probing and constraining new physics.

B. CP violation in $K \to \pi\nu\bar{\nu}$

Observing CP violation in the rare $K \to \pi\nu\bar{\nu}$ decays would be experimentally very challenging and theoretically very rewarding. It is very different from the CP violation that has been observed in $K \to \pi\pi$ decays which is small and involves theoretical uncertainties. Similar to the CP asymmetry in $B \to \psi K_S$, it is predicted to be large and can be cleanly interpreted. Furthermore, observation of the $K_L \to \pi^0\nu\bar{\nu}$ decay at the rate predicted by the Standard Model will provide further evidence that CP violation cannot be attributed to mixing ($\Delta S = 2$) processes only, as in superweak models.

Define

$$A_{\pi^0\nu\bar{\nu}} = \langle \pi^0\nu\bar{\nu}|\mathcal{H}|K^0\rangle, \quad \bar{A}_{\pi^0\nu\bar{\nu}} = \langle \pi^0\nu\bar{\nu}|\mathcal{H}|\bar{K}^0\rangle, \quad \lambda_{\pi\nu\bar{\nu}} = \left(\frac{q}{p}\right)_K \frac{\bar{A}_{\pi^0\nu\bar{\nu}}}{A_{\pi^0\nu\bar{\nu}}}. \tag{4.19}$$

The ratio between the neutral K decay rates is then

$$\frac{\Gamma(K_L \to \pi^0\nu\bar{\nu})}{\Gamma(K_S \to \pi^0\nu\bar{\nu})} = \frac{1 + |\lambda_{\pi\nu\bar{\nu}}|^2 - 2\mathrm{Re}\lambda_{\pi\nu\bar{\nu}}}{1 + |\lambda_{\pi\nu\bar{\nu}}|^2 + 2\mathrm{Re}\lambda_{\pi\nu\bar{\nu}}}. \tag{4.20}$$

We learn that the $K_L \to \pi^0\nu\bar{\nu}$ decay rate vanishes in the CP limit ($\lambda_{\pi\nu\bar{\nu}} = 1$), as expected on general grounds [55]. (The CP conserving contributions were explicitly calculated within the Standard Model [56] and within its extensions with massive neutrinos [57] and with extra scalars [58] and found to be negligible.)

CP violation in decay and in mixing are expected to be negligibly small, of order 10^{-5} and 10^{-3}, respectively. Consequently, $\lambda_{\pi\nu\bar{\nu}}$ is, to an excellent approximation, a pure phase. Defining $2\theta_K$ to be the relative phase between the $K - \bar{K}$ mixing amplitude and twice the $s \to d\nu\bar{\nu}$ decay amplitude, namely $\lambda_{\pi\nu\bar{\nu}} = e^{2i\theta_K}$, we get from (4.20):

$$\frac{\Gamma(K_L \to \pi^0 \nu\bar{\nu})}{\Gamma(K_S \to \pi^0 \nu\bar{\nu})} = \tan^2 \theta_K. \tag{4.21}$$

Using the isospin relation $A(K^0 \to \pi^0 \nu\bar{\nu})/A(K^+ \to \pi^+ \nu\bar{\nu}) = 1/\sqrt{2}$, we get

$$a_{\pi\nu\bar{\nu}} \equiv \frac{\Gamma(K_L \to \pi^0 \nu\bar{\nu})}{\Gamma(K^+ \to \pi^+ \nu\bar{\nu})} = \sin^2 \theta_K. \tag{4.22}$$

The present experimental searches give

$$\begin{aligned} \mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) &= (1.5^{+3.4}_{-1.2}) \times 10^{-10} \quad [59], \\ \mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) &< 5.9 \times 10^{-7} \quad [60]. \end{aligned} \tag{4.23}$$

Eq. (4.22) implies that $a_{\pi\nu\bar{\nu}} \leq 1$. This inequality is based on isospin considerations only. Consequently a measurement of $\Gamma(K^+ \to \pi^+ \nu\bar{\nu})$ can be used to set a model independent upper limit on $\Gamma(K_L \to \pi^0 \nu\bar{\nu})$ [61]:

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) \; < \; 4.4 \, \mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}). \tag{4.24}$$

From the range in (4.23) of the K^+ decay, the isospin bound on the K_L decay is $\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) < 2.6 \times 10^{-9}$, which is more than two orders of magnitude below the direct bound.

Within the Standard Model, the $K \to \pi\nu\bar{\nu}$ decays are dominated by short distance Z-penguins and box diagrams and can be expressed in terms of ρ and η (see [53] for details and references)

$$\begin{aligned} \mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) &= 4.11 \times 10^{-11} [X(x_t)]^2 A^4 \left[\eta^2 + (\rho_0 - \rho)^2\right], \\ \mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) &= 1.80 \times 10^{-10} [X(x_t)]^2 A^4 \eta^2. \end{aligned} \tag{4.25}$$

Here $\rho_0 = 1 + \frac{P_0(X)}{A^2 X(x_t)}$, and $X(x_t)$ and $P_0(X)$ represent the electroweak loop contributions in NLO for the top quark and for the charm quark, respectively.

We would like to emphasize the following points:

(i) The $K \to \pi\nu\bar{\nu}$ decays are theoretically clean. The main theoretical uncertainty in the K^+ decay is related to the strong dependence of the charm contribution on the renormalization scale and the QCD scale, $P_0(X) = 0.42 \pm 0.06$. The K_L decay has hadronic uncertainties smaller than a percent.

(ii) In the future, these decays will provide excellent $\rho - \eta$ constraints.

(iii) Present constraints on the CKM parameters give the SM predictions [47]:

$$\begin{aligned} \mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) &= (7.0 \pm 1.9) \times 10^{-11}, \\ \mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) &= (2.9 \pm 1.1) \times 10^{-11}. \end{aligned} \tag{4.26}$$

The experimental range for the K^+ decay (4.23) is then consistent with the SM but not yet accurate enough to constrain it, while the experimental bound on the K_L decay is still four orders of magnitude above the SM range.

(iv) The CP violations in $K \to \pi\nu\bar{\nu}$ and in $B \to \psi K_S$ are strongly correlated and can provide the most stringent test of the Kobayashi-Maskawa mechanism.

(v) The $K \to \pi\nu\bar{\nu}$ decays are interesting probes of CP violation related to new physics.

V. D DECAYS

Within the Standard Model, $D-\bar{D}$ mixing is expected to be well below the experimental bound. Furthermore, effects related to CP violation in $D-\bar{D}$ mixing are expected to be negligibly small since this mixing is described to an excellent approximation by physics of the first two generations. An experimental observation of $D-\bar{D}$ mixing close to the present bound or, more strongly, of related CP violation, will then be evidence for New Physics.

To explain how $D-\bar{D}$ mixing is searched for and how CP violation can be signalled, we use notations similar to those of the B system. We thus use eq. (3.2) to define the two mass eigenstates $|D_{1,2}\rangle$, eq. (3.5) to define the average width Γ, eq. (3.6) to define the width and mass differences y and x, eq. (3.15) to define the decay amplitudes A_f and $\bar{A}_f$ and eq. (3.17) to define λ_f.

A. $D\to K\pi$ and $D\to KK$ Decays

The processes that are relevant to the most sensitive measurements at present are the doubly-Cabibbo-suppressed $D^0\to K^+\pi^-$ decay, the singly-Cabibbo-suppressed $D^0\to K^+K^-$ decay, the Cabibbo-favored $D^0\to K^-\pi^+$ decay, and the three CP-conjugate decay processes. We follow here the analysis presented in ref. [62]. We write down approximate expressions for the time-dependent decay rates that are valid for times $t\lesssim 1/\Gamma$. We take into account the experimental information that x, y and $\tan\theta_c$ are small. In particular, the smallness of $\tan\theta_c$ implies that

$$|\lambda^{-1}_{K^+\pi^-}|\ll 1;\quad |\lambda_{K^-\pi^+}|\ll 1. \tag{5.1}$$

We expand each of the rates only to the order that is relevant to present measurements:

$$\begin{aligned}
\Gamma[D^0(t)\to\ K^+\pi^-] &= e^{-\Gamma t}|\bar{A}_{K^+\pi^-}|^2|q/p|^2\\
&\times\left\{|\lambda^{-1}_{K^+\pi^-}|^2+[\mathrm{Re}(\lambda^{-1}_{K^+\pi^-})y+\mathrm{Im}(\lambda^{-1}_{K^+\pi^-})x]\Gamma t+\frac{1}{4}(y^2+x^2)(\Gamma t)^2\right\},\\
\Gamma[\overline{D^0}(t)\to\ K^-\pi^+] &= e^{-\Gamma t}|A_{K^-\pi^+}|^2|p/q|^2\\
&\times\left\{|\lambda_{K^-\pi^+}|^2+[\mathrm{Re}(\lambda_{K^-\pi^+})y+\mathrm{Im}(\lambda_{K^-\pi^+})x]\Gamma t+\frac{1}{4}(y^2+x^2)(\Gamma t)^2\right\},
\end{aligned}\tag{5.2}$$

$$\begin{aligned}
\Gamma[D^0(t)\to K^+K^-] &= e^{-\Gamma t}|A_{K^+K^-}|^2\{1+[\mathrm{Re}(\lambda_{K^+K^-})y-\mathrm{Im}(\lambda_{K^+K^-})x]\Gamma t\},\\
\Gamma[\overline{D^0}(t)\to K^+K^-] &= e^{-\Gamma t}|\bar{A}_{K^+K^-}|^2\left\{1+[\mathrm{Re}(\lambda^{-1}_{K^+K^-})y-\mathrm{Im}(\lambda^{-1}_{K^+K^-})x]\Gamma t\right\},
\end{aligned}\tag{5.3}$$

$$\begin{aligned}
\Gamma[D^0(t)\to\ K^-\pi^+] &= e^{-\Gamma t}|A_{K^-\pi^+}|^2,\\
\Gamma[\overline{D^0}(t)\to\ K^+\pi^-] &= e^{-\Gamma t}|\bar{A}_{K^+\pi^-}|^2.
\end{aligned}\tag{5.4}$$

Within the Standard Model, the physics of $D^0-\overline{D^0}$ mixing and of the tree level decays is dominated by the first two generations and, consequently, CP violation can be safely neglected. In almost all 'reasonable' extensions of the SM, the six decay modes of eqs. (5.2), (5.3) and (5.4) are still dominated by the SM CP conserving contributions [63,64]. On the other hand, there could be new short distance, possibly CP violating contributions to the mixing amplitude M_{12}. Allowing for only such effects of new physics, the picture of

CP violation is simplified since there is no direct CP violation. The effects of indirect CP violation can be parameterized in the following way [65]:

$$
\begin{aligned}
|q/p| &= R_m, \\
\lambda^{-1}_{K^+\pi^-} &= \sqrt{R}\ R_m^{-1}\ e^{-i(\delta+\phi_D)}, \\
\lambda_{K^-\pi^+} &= \sqrt{R}\ R_m\ e^{-i(\delta-\phi_D)}, \\
\lambda_{K^+K^-} &= -R_m\ e^{i\phi_D}.
\end{aligned} \tag{5.5}
$$

Here R and R_m are real and positive dimensionless numbers. CP violation in mixing is related to $R_m \neq 1$ while CP violation in the interference of decays with and without mixing is related to $\sin\phi_D \neq 0$. The choice of phases and signs in (5.5) is consistent with having $\phi_D = 0$ in the SM and $\delta = 0$ in the $SU(3)$ limit. We further define

$$
\begin{aligned}
x' &\equiv x\cos\delta + y\sin\delta, \\
y' &\equiv y\cos\delta - x\sin\delta.
\end{aligned} \tag{5.6}
$$

With our assumption that there is no direct CP violation in the processes that we study, and using the parametrizations (5.5) and (3.6), we can rewrite eqs. (5.2), (5.3) and (5.4) as follows:

$$
\begin{aligned}
\Gamma[D^0(t) \to\ K^+\pi^-] \ &=\ e^{-\Gamma t}|A_{K^-\pi^+}|^2 \\
&\times \left[R + \sqrt{R}R_m(y'\cos\phi_D - x'\sin\phi_D)\Gamma t + \frac{R_m^2}{4}(y^2+x^2)(\Gamma t)^2\right], \\
\Gamma[\overline{D^0}(t) \to\ K^-\pi^+] \ &=\ e^{-\Gamma t}|A_{K^-\pi^+}|^2 \\
&\times \left[R + \sqrt{R}R_m^{-1}(y'\cos\phi_D + x'\sin\phi_D)\Gamma t + \frac{R_m^{-2}}{4}(y^2+x^2)(\Gamma t)^2\right],
\end{aligned} \tag{5.7}
$$

$$
\begin{aligned}
\Gamma[D^0(t) \to\ K^+K^-] \ &=\ e^{-\Gamma t}|A_{K^+K^-}|^2\left[1 - R_m(y\cos\phi_D - x\sin\phi_D)\Gamma t\right], \\
\Gamma[\overline{D^0}(t) \to\ K^+K^-] \ &=\ e^{-\Gamma t}|A_{K^+K^-}|^2\left[1 - R_m^{-1}(y\cos\phi_D + x\sin\phi_D)\Gamma t\right],
\end{aligned} \tag{5.8}
$$

$$
\Gamma[D^0(t) \to K^-\pi^+] \ =\ \Gamma[\overline{D^0}(t) \to K^+\pi^-] \ =\ e^{-\Gamma t}|A_{K^-\pi^+}|^2. \tag{5.9}
$$

Of particular interest is the linear term in eq. (5.7) which is potentially CP violating [66,67]. It is useful to define a CP violating quantity $a_{D\to K\pi}$ which depends on the six measurable coefficients in (5.7):

$$
\begin{aligned}
a_{D\to K\pi} &= \frac{\mathrm{Re}(\lambda_{K^-\pi^+})y + \mathrm{Im}(\lambda_{K^-\pi^+})x}{2|\lambda_{K^-\pi^+}|\sqrt{x^2+y^2}} - \frac{\mathrm{Re}(\lambda^{-1}_{K^+\pi^-})y + \mathrm{Im}(\lambda^{-1}_{K^+\pi^-})x}{2|\lambda^{-1}_{K^+\pi^-}|\sqrt{x^2+y^2}} \\
&= \frac{x'}{\sqrt{x^2+y^2}}\sin\phi_D.
\end{aligned} \tag{5.10}
$$

Observing $a_{D\to K\pi} \neq 0$ would be the most convincing evidence for new physics in $D - \bar{D}$ mixing.

The CLEO measurement [68] gives the coefficient of each of the three terms [1, Γt and $(\Gamma t)^2$] in the doubly-Cabibbo suppressed decays (5.7). Such measurements allow a fit to the parameters R, R_m, $x'\sin\phi$, $y'\cos\phi$, and x^2+y^2. Fit A of ref. [68] quotes the following one sigma ranges:

$$\begin{aligned} R &= (0.48 \pm 0.13) \times 10^{-2}, \\ y' \cos\phi_D &= (-2.5^{+1.4}_{-1.6}) \times 10^{-2}, \\ x' &= (0.0 \pm 1.5) \times 10^{-2}, \\ A_m &= 0.23^{+0.63}_{-0.80}, \\ \sin\phi_D &= 0.0 \pm 0.6. \end{aligned} \tag{5.11}$$

It is assumed here that R_m is not very different from one and can be parameterized by a small parameter A_m,

$$R_m^{\pm 2} = 1 \pm A_m. \tag{5.12}$$

We would like to make two further comments in this regard:

(i) The experimental results in eq. (5.11) do not show any signal of CP violation, that is, both $\sin\phi_D$ and A_m are consistent with zero. Consequently, there is no hint of new physics in the present results.

(ii) To test models of new physics, it would be useful to know the value of the strong phase δ. Such an estimate is a difficult theoretical task [69–71] but experimental data on related channels would be useful [72,73].

As concerns the singly-Cabibbo suppressed modes (5.8), several experiments fit the time dependent decay rates to pure exponentials. We define $\hat\Gamma$ to be the parameter that is extracted in this way. More explicitly, for a time dependent decay rate with $\Gamma[D(t) \to f] \propto e^{-\Gamma t}(1 - z\Gamma t + \cdots)$, where $|z| \ll 1$, we have $\hat\Gamma(D \to f) = \Gamma(1+z)$. The above equations imply the following relations:

$$\begin{aligned} \hat\Gamma(D^0 \to K^+K^-) &= \Gamma\ [1 + R_m(y\cos\phi_D - x\sin\phi_D)], \\ \hat\Gamma(\overline{D^0} \to K^+K^-) &= \Gamma\ [1 + R_m^{-1}(y\cos\phi_D + x\sin\phi_D)], \\ \hat\Gamma(D^0 \to K^-\pi^+) &= \hat\Gamma(\overline{D^0} \to K^+\pi^-) = \Gamma. \end{aligned} \tag{5.13}$$

Note that deviations of $\hat\Gamma(D \to K^+K^-)$ from Γ do not require that $y \neq 0$. They can be accounted for by $x \neq 0$ and $\sin\phi_D \neq 0$, but then they have a different sign in the D^0 and $\overline{D^0}$ decays. Combining the two $D \to K^+K^-$ modes, one obtains the CP conserving quantity $y_{\rm CP}$:

$$\begin{aligned} y_{\rm CP} &\equiv \frac{\hat\Gamma(D \to K^+K^-)}{\hat\Gamma(D^0 \to K^-\pi^+)} - 1 \\ &= y\cos\phi_D - \frac{A_m}{2} x \sin\phi, \end{aligned} \tag{5.14}$$

where we made the approximations of zero production asymmetry and small A_m [62]. The one sigma ranges measured by various experiments are given by

$$y_{\rm CP} = \begin{cases} (3.4 \pm 1.6) \times 10^{-2} & \text{FOCUS [74]} \\ (0.8 \pm 3.1) \times 10^{-2} & \text{E791 [75]} \\ (-1.1 \pm 2.9) \times 10^{-2} & \text{CLEO [76]} \\ (0.5 \pm 1.3) \times 10^{-2} & \text{BELLE [77]} \end{cases} \tag{5.15}$$

giving a world average of

$$y_{\rm CP} = (1.3 \pm 0.9) \times 10^{-2}. \tag{5.16}$$

Finally, we note that direct CP violation has been searched for in the Cabibbo-favored [78], singly-Cabibbo-suppressed [79–81] and doubly-Cabibbo-suppressed [68] decays with all results consistent with zero.

We conclude that at present there is no evidence for mixing and certainly not for CP violation in the neutral D system. These results are consistent with the SM and constrain models of new physics. If evidence is found in the future, the $D \to K\pi$ and $D \to KK$ decays will provide rich enough information that we will be able to point out the origin of the signals in much detail.

VI. B DECAYS

A. CP Violation in Mixing

CP violation in mixing is related to a non-zero value for the following quantity [see eq. (3.13)]:

$$1 - \left|\frac{q}{p}\right| \simeq \frac{1}{2}{\rm Im}\left(\frac{\Gamma_{12}}{M_{12}}\right). \tag{6.1}$$

The effect can be isolated by measuring the asymmetry in semileptonic decays [see eq. (3.26)]:

$$a_{\rm SL} \simeq 2(1 - |q/p|) \simeq {\rm Im}(\Gamma_{12}/M_{12}). \tag{6.2}$$

This has been searched for in several experiments, with sensitivity at the level of 10^{-2}:

$$a_{\rm SL} = \begin{cases} (1.4 \pm 4.2) \times 10^{-2} & \text{CLEO [82]} \\ (0.4 \pm 5.7) \times 10^{-2} & \text{OPAL [83]} \\ (-1.2 \pm 2.8) \times 10^{-2} & \text{ALEPH [84]} \\ (0.48 \pm 1.85) \times 10^{-2} & \text{BABAR [85]} \end{cases} \tag{6.3}$$

giving a world average of

$$a_{\rm SL} = (0.2 \pm 1.4) \times 10^{-2}. \tag{6.4}$$

As explained above, in the B_d system we expect model independently that $|\Gamma_{12}/M_{12}| \ll 1$. Within any given model we can actually calculate the two quantities from quark diagrams. Within the SM, M_{12} is given by box diagrams. For both the B_d and B_s systems, the long distance contributions are expected to be negligible and the calculation of these diagrams with a high loop momentum is a very good approximation. Γ_{12} is calculated from a cut of box diagrams [86]. Since the cut of a diagram always involves on-shell particles and thus long distance physics, the calculation is, at best, a reasonable approximation to Γ_{12}. (For $\Gamma_{12}(B_s)$ it has been shown that local quark-hadron duality holds exactly in the simultaneous limit of small velocity and large number of colors. We thus expect an uncertainty of $\mathcal{O}(1/N_C) \sim 30\%$ [87,88]. For $\Gamma_{12}(B_d)$ the small velocity limit is not as good an approximation but an uncertainty of order 50% still seems a reasonable estimate [89].)

Within the Standard Model, M_{12} is dominated by top-mediated box diagrams (see [53] for details and references):

$$M_{12} = \frac{G_F^2}{12\pi^2} m_B m_W^2 \eta_B B_B f_B^2 (V_{tb}V_{td}^*)^2 S_0(x_t), \tag{6.5}$$

where $S_0(x_t)$ is a kinematic factor, η_B is a QCD correction, and $B_B f_B^2$ parametrizes the hadronic matrix element. For Γ_{12}, we have [91–93]

$$\begin{aligned} \Gamma_{12} &= -\frac{G_F^2}{24\pi} m_B m_b^2 B_B f_B^2 (V_{tb}V_{td}^*)^2 \\ &\times \left[\frac{5}{3}\frac{m_B^2}{(m_b+m_d)^2}\frac{B_S}{B_B}(K_2-K_1) + \frac{4}{3}(2K_1+K_2) + 8(K_1+K_2)\frac{m_c^2}{m_b^2}\frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*} \right], \end{aligned} \tag{6.6}$$

where $K_1 = -0.39$ and $K_2 = 1.25$ [93] are combinations of Wilson coefficients and B_S parametrizes the $(S-P)^2$ matrix element. New physics usually takes place at a high energy scale and is relevant to the short distance part only. Therefore, the SM estimate in eq. (6.6) remains valid model independently. Combining (6.5) and (6.6), we learn that $|\Gamma_{12}/M_{12}| = \mathcal{O}(m_b^2/m_t^2)$, which confirms our model independent order of magnitude estimate, $|\Gamma_{12}/M_{12}| \lesssim 10^{-2}$. As concerns the imaginary part of this ratio, we have

$$a_{\rm SL} = {\rm Im}\frac{\Gamma_{12}}{M_{12}} \approx -1.4 \times 10^{-3} \frac{\eta}{(1-\rho)^2+\eta^2}. \tag{6.7}$$

The suppression by a factor of $\mathcal{O}(10)$ of $a_{\rm SL}$ compared to $|\Gamma_{12}/M_{12}|$ comes from the fact that the leading contribution to Γ_{12} has the same phase as M_{12}. Consequently, $a_{\rm SL} = \mathcal{O}(m_c^2/m_t^2)$. The CKM factor does not give any further significant suppression, ${\rm Im}\frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*} = \mathcal{O}(1)$. In contrast, for the B_s system, where the same expressions holds except that V_{cd}/V_{td} is replaced by V_{cs}/V_{ts}, there is an additional CKM suppression from ${\rm Im}\frac{V_{cb}V_{cs}^*}{V_{tb}V_{ts}^*} = \mathcal{O}(\lambda^2)$.

In the SM and in most of its reasonable extensions, both Γ_{12} and $b \to c\bar{c}s$ transitions are dominated by SM tree level decays. Consequently, new physics affects $a_{\rm SL}$ and $a_{\psi K_S}$ only through its contributions to M_{12}. This leads to interesting correlations between $a_{\rm SL}$ and $a_{\psi K_S}$ that can be used to probe flavor parameters [94,95]. Conversely, one can use the measured value of $a_{\psi K_S}$ to give model independent predictions for $a_{\rm SL}$ [96,97].

B. Penguin Pollution

In purely hadronic B decays, CP violation in decay and in the interference of decays with and without mixing is $\geq \mathcal{O}(10^{-2})$. We can therefore safely neglect CP violation in mixing in the following discussion and use

$$\frac{q}{p} = \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\omega_B. \tag{6.8}$$

(From here on we omit the convention-dependent quark phases ω_q defined in eq. (3.18). Our final expressions for physical quantities are of course unaffected by such omission.)

A crucial aspect of our discussion is the number of relevant weak phases for a given decay process:

(i) If there is a single weak phase that dominates the decay, CP violation in decay will be small and difficult to observe. On the other hand, CP asymmetries in neutral B decays into final CP eigenstates are subject to clean theoretical interpretation: we will either have precise measurements of CKM parameters or be provided with unambiguous evidence for new physics.

(ii) If there are two (or more) weak phases that contribute comparably, hadronic uncertainties will appear in the theoretical interpretation of CP violation in the interference of decays with and without mixing. On the other hand, if there are also large strong phase differences, CP violation in decay can be observed in the corresponding charged and neutral B decays.

In many cases of interest, different weak phases are carried by tree and penguin contributions. The difficulties arising from hadronic uncertainties related to comparable tree and penguin contributions became known as "penguin pollution."

To illustrate the problem, we will consider two relevant CP asymmetries. First, the CP asymmetry in $B \to \psi K_S$ is an example of a case where the penguin pollution is negligibly small and a theoretically very clean interpretation of the experimental measurement is possible. Second, the CP asymmetry in $B \to \pi\pi$ is an example of a case where penguin pollution cannot be a-priori ignored. We also list various ways in which the problem might be overcome.

C. $B \to \psi K_S$

The first evidence for CP violation outside K decays has been provided by the recent BaBar and Belle measurements of the CP asymmetry in $B \to \psi K_S$,

$$a_{\psi K_S} = \begin{cases} 0.59 \pm 0.15 & \text{Babar [11]} \\ 0.99 \pm 0.15 & \text{Belle [12]} \end{cases} \tag{6.9}$$

These results in combination with previous ones [8–10] give the world average quoted in eq. (1.3). The process $B \to \psi K_S$ is one where the penguin contribution is harmless and the CP asymmetry is subject to an impressingly clean theoretical interpretation.

The decay is mediated by the quark transition $\bar{b} \to \bar{c}c\bar{s}$. It gets contributions from a tree level diagram and from penguin diagrams with intermediate u, c and t quarks. Using the unitarity relation (2.46), we can write the various contributions in terms of two CKM combinations:

$$A(\bar{b} \to \bar{c}c\bar{s}) = (T_{c\bar{c}s} + P_s^c - P_s^t) V_{cb}^* V_{cs} + (P_s^u - P_s^t) V_{ub}^* V_{us}. \tag{6.10}$$

The second term is suppressed by two factors. First, there is the ratio between penguin and tree contributions,

$$r_{PT}^{\psi K} \equiv \frac{P_{\psi K}}{T_{\psi K}} \equiv \frac{P_s^u - P_s^t}{T_{c\bar{c}s} + P_s^c - P_s^t}$$
$$\approx \left[\frac{\alpha_s}{12\pi} \ln \frac{m_t^2}{m_b^2}\right] \frac{\langle \psi K_S | \bar{b}\gamma^\mu T^a s \bar{c}\gamma_\mu T^a c | B^0\rangle}{\langle \pi^+\pi^- | \bar{b}_L \gamma^\mu c_L \bar{c}_L \gamma_\mu s_L | B^0\rangle}. \tag{6.11}$$

The term is brackets is $\mathcal{O}(0.03)$ but the ratio of matrix elements may partially compensate for this suppresion. Second, there is the ratio of CKM elements, $|(V_{ub}^* V_{us})/(V_{cb}^* V_{cs})| \sim \lambda^2$. We conclude that the second term is suppressed by $r_{PT}^{\psi K}\lambda^2 \lesssim 10^{-2}$ and we can safely neglect $P_{\psi K_S}$. Thus the $B \to \psi K$ decay is dominated by a single weak phase, that is, $\arg(V_{cb}^* V_{cs})$.

Nelecting $P_{\psi K_S}$ means that, to a very good approximation, we have $|\lambda_{\psi K_S}| = 1$,

$$a_{\psi K_S} = \mathrm{Im}\lambda_{\psi K_S}, \tag{6.12}$$

and that the experimental value of $a_{\psi K_S}$ [eq. (1.3)] can be cleanly interpreted in terms of a CP violating phase.

A new ingredient in the analysis is the effect of $K - \bar{K}$ mixing. For decays with a single K_S in the final state, $K - \bar{K}$ mixing is essential because $B^0 \to K^0$ and $\bar{B}^0 \to \bar{K}^0$, and interference is possible only due to $K - \bar{K}$ mixing. This adds a factor of

$$\left(\frac{p}{q}\right)_K = \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} \omega_K^* \tag{6.13}$$

into $(\bar{A}/A)$:

$$\frac{\bar{A}_{\psi K_S}}{A_{\psi K_S}} = \eta_{\psi K_S} \left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\right) \left(\frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}\right) \omega_B^*. \tag{6.14}$$

The CP-eigenvalue of the state is $\eta_{\psi K_S} = -1$. Combining eqs. (6.8) and (6.14), we find

$$\lambda(B \to \psi K_S) = -\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right) \left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\right) \left(\frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}\right), \tag{6.15}$$

which leads to

$$a_{\psi K_S} = \sin 2\beta. \tag{6.16}$$

What we have learnt above is that eq. (6.16) is clean of hadronic uncertainties to $\mathcal{O}(r_{PT}^{\psi K}\lambda^2) \lesssim 10^{-2}$. This means that the measuremnet of $a_{\psi K_S}$ can give the theoretically cleanest determination of a CKM parameter, even cleaner than the determination of $|V_{us}|$ from $K \to \pi \ell \nu$. [If $\mathrm{BR}(K_L \to \pi \nu \bar{\nu})$ is measured, it will give a comparably clean determination of η.]

Taking into account all the constraints on the CKM parameters *except* for the $a_{\psi K_S}$ measurements, the SM prediction is [47]

$$\sin 2\beta = 0.68 \pm 0.18, \tag{6.17}$$

consistent with the experimental result (1.3). This consistency has important implications. In particular,

(i) The Kobayashi-Maskawa mechanism has successfully passed its first precision test;

(ii) Models of approximate CP which, by definition, predict $|a_{\psi K_S}| \ll 1$, are excluded.

D. $B \to \pi\pi$

The CP asymmetry in the $B \to \pi^+\pi^-$ mode has the form

$$a_{\pi\pi}(t) = -\frac{1-|\lambda_{\pi\pi}|^2}{1+|\lambda_{\pi\pi}|^2}\cos\Delta mt + \frac{2\mathrm{Im}\lambda_{\pi\pi}}{1+|\lambda_{\pi\pi}|^2}\sin\Delta mt. \tag{6.18}$$

Recently, the BaBar collaboration presented the first constraints on this asymmetry [99]:

$$\begin{aligned}\frac{2\mathrm{Im}\lambda_{\pi\pi}}{1+|\lambda_{\pi\pi}|^2} &= 0.03^{+0.54}_{-0.57},\\ \frac{1-|\lambda_{\pi\pi}|^2}{1+|\lambda_{\pi\pi}|^2} &= -0.25^{+0.47}_{-0.49}.\end{aligned} \tag{6.19}$$

The results are not yet precise enough to give useful constraints. But we discuss this mode to show how penguin pollution arises and how it complicates the analysis.

The decay is mediated by the quark transition $\bar{b} \to \bar{u}u\bar{d}$. It gets contributions from a tree level diagram and from penguin diagrams with intermediate u, c and t quarks. Using the unitarity relation (2.47), we can write the various contributions in terms of two CKM combinations:

$$A(\bar{b} \to \bar{u}u\bar{d}) = (T_{u\bar{u}d} + P_d^u - P_d^c)V_{ub}^*V_{ud} + (P_d^t - P_d^c)V_{tb}^*V_{td}. \tag{6.20}$$

The ratio between the magnitudes of the second and first terms is given by $r_{PT}^{\pi\pi}\left|\frac{V_{tb}^*V_{td}}{V_{ub}^*V_{ud}}\right|$. Since both $|V_{ub}V_{ud}^*|$ and $|V_{tb}^*V_{td}|$ are of $\mathcal{O}(\lambda^3)$, the second term is suppressed only by the factor $r_{PT}^{\pi\pi}$, where

$$r_{PT}^{\pi\pi} \equiv \frac{P_{\pi\pi}}{T_{\pi\pi}} \equiv \frac{P_d^t - P_d^c}{T_{u\bar{u}d} + P_d^u - P_d^c}. \tag{6.21}$$

One may make a rough estimate of $|P_{\pi\pi}/T_{\pi\pi}|$ from the decay $B \to K\pi$, which can be parameterized as follows:

$$A(B^0 \to K^+\pi^-) = T_{K\pi}V_{ub}^*V_{us} + P_{K\pi}V_{tb}V_{ts}^*. \tag{6.22}$$

In this case $|P_{K\pi}/T_{K\pi}| = \mathcal{O}(r_{PT}^{K\pi}/\lambda^2)$. If QCD enhances the penguin contribution to $B \to \pi\pi$ by a significant amount, that is, $r_{PT} \gg \lambda^2$, then $B \to K\pi$ would be dominated by the penguin process. Let us provisionally make the following assumptions: (i) flavor SU(3) symmetry in the QCD matrix elements; (ii) electroweak penguins and "color suppressed" processes are negligible; (iii) penguins dominate $B \to K\pi$, so $T_{K\pi}$ may be ignored in $\mathrm{BR}(B^0 \to K^+\pi^-)$; (iv) penguins make a small enough contribution to $B \to \pi\pi$ that $P_{\pi\pi}$ may be ignored in $\mathrm{BR}(B^0 \to \pi^+\pi^-)$. Then

$$\left|\frac{P_{\pi\pi}}{T_{\pi\pi}}\right| = \left|\frac{P_{\pi\pi}}{P_{K\pi}}\right|\left|\frac{P_{K\pi}}{T_{\pi\pi}}\right| = \left|\frac{V_{ub}V_{ud}}{V_{ts}V_{tb}}\right|\sqrt{\frac{\mathrm{BR}(B^0 \to K^+\pi^-)}{\mathrm{BR}(B^0 \to \pi^+\pi^-)}}. \tag{6.23}$$

Recent measurements [100–102] give world averages $\mathrm{BR}(B^0 \to \pi^+\pi^-) = (4.4 \pm 0.9) \times 10^{-6}$ and $\mathrm{BR}(B^0 \to K^+\pi^-) = (17.3 \pm 1.5) \times 10^{-6}$. We thus find $\mathrm{BR}(B^0 \to K^+\pi^-)/\mathrm{BR}(B^0 \to \pi^+\pi^-) \approx 3.9$ and obtain the rough estimate

$$|r_{PT}^{\pi\pi}| \sim 0.2 - 0.3. \tag{6.24}$$

It is clear that penguin effects are unlikely to be negligible in $B \to \pi\pi$.

Combining eqs. (6.8) and (6.20), we find

$$\lambda(B \to \pi\pi) = \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)\left(\frac{V_{ub} V_{ud}^*}{V_{ub}^* V_{ud}}\right)\left[\frac{1 + r_{PT}^{\pi\pi}(V_{tb} V_{td}^*)/(V_{ub} V_{ud}^*)}{1 + r_{PT}^{\pi\pi}(V_{tb}^* V_{td})/(V_{ub}^* V_{ud})}\right]. \tag{6.25}$$

If the last factor could be approximated by unity, that is, $r_{PT}^{\pi\pi} = 0$, we would obtain $|\lambda_{\pi\pi}| = 1$ and

$$a_{\pi\pi} = \sin 2\alpha. \tag{6.26}$$

This approximation is however unjustified. To get an idea of the effects of $P_{\pi\pi} \neq 0$, we give the leading corrections due to a small $|r_{PT}|$:

$$\begin{aligned} |\lambda_{\pi\pi}| &= 1 - 2(R_t/R_u)\mathrm{Im}(r_{PT}^{\pi\pi}) \sin\alpha, \\ \mathrm{Im}\lambda_{\pi\pi}/|\lambda_{\pi\pi}| &= \sin 2\alpha + 2(R_t/R_u)\mathrm{Re}(r_{PT}^{\pi\pi}) \cos 2\alpha \sin\alpha. \end{aligned} \tag{6.27}$$

(For a more detailed discussion, see [98].) Note that if strong phases can be neglected, r_{PT} is real and $|\lambda_{\pi\pi}| = 1$ would be a good approximation. But it is not clear whether the strong phases are indeed small. In any case, one needs to know $r_{PT}^{\pi\pi}$ to extract α from $a_{\pi\pi}(t)$. This is the problem of the penguin pollution.

A variety of solutions to this problem have been proposed, falling roughly into two classes. The first type of approach is to convert the estimate given above into an actual measurement of $|P_{K\pi}|$. (The list of papers on this subject is long. Early works include [103–105]. For a much more comprehensive list of references, see [98].) Once $|P_{K\pi}|$ is known, flavor $SU(3)$ is used to relate $|P_{K\pi}|$ to $|P_{\pi\pi}|$. One must then include a number of additional effects:

(i) Electroweak penguins. The effects are calculable [106].

(ii) Color suppressed and rescattering processes. These must be bounded or estimated using data and some further assumptions.

(iii) $SU(3)$ corrections. Some, such as f_K/f_π, can be included, but $SU(3)$ corrections generally remain a source of irreducible uncertainty.

The second type of approach is to exploit the fact that the penguin contribution to $P_{\pi\pi}$ is pure $\Delta I = \frac{1}{2}$, while the tree contribution to $T_{\pi\pi}$ contains a piece which is $\Delta I = \frac{3}{2}$. (This is not true of the electroweak penguins [107], but these are expected to be small.) Isospin symmetry allows one to form a relation among the amplitudes $B^0 \to \pi^+\pi^-$, $B^0 \to \pi^0\pi^0$, and $B^+ \to \pi^+\pi^0$,

$$\frac{1}{\sqrt{2}} A(B^0 \to \pi^+\pi^-) + A(B^0 \to \pi^0\pi^0) = A(B^+ \to \pi^+\pi^0). \tag{6.28}$$

There is also a relation for the charge conjugate processes. A simple geometric construction then allows one to disentangle the unpolluted $\Delta I = \frac{3}{2}$ amplitudes, from which $\sin 2\alpha$ may be extracted cleanly [108].

The key experimental difficulty is that one must measure accurately the flavor-tagged rate for $B^0 \to \pi^0\pi^0$. Since the final state consists of only four photons, and the branching fraction is expected to be of $\mathcal{O}(10^{-6})$, this is very hard. It has been noted that an upper bound on this rate, if sufficiently strong, would also allow one to bound $P_{\pi\pi}$ usefully [109,98,110].

An alternative is to perform an isospin analysis of the process $B^0 \to \rho\pi \to \pi^+\pi^-\pi^0$ [111–114]. Here one must study the time-dependent asymmetry over the entire Dalitz plot, probing variously the intermediate states $\rho^\pm\pi^\mp$ and $\rho^0\pi^0$. The advantage here is that the final states with two π^0's need not be considered. On the other hand, thousands of cleanly reconstructed events would be needed.

Finally, one might attempt to calculate the penguin matrix elements. Model-dependent analyses are not really adequate for this purpose, since the goal is the extraction of fundamental parameters. Precise calculations of such matrix elements from lattice QCD are far in the future, given the large energies of the π's and the need for an unquenched treatment. Recently, a new QCD-based analysis of the $B \to \pi\pi$ matrix elements has been proposed [115–118]. For details, see [119].

VII. CP VIOLATION IN SUPERSYMMETRY

A. CP Violation as a Probe of New Physics

We have argued that the Standard Model picture of CP violation is rather unique and highly predictive. We have also stated that reasonable extensions of the Standard Model have a very different picture of CP violation. Experimental results are now starting to decide between the various possibilities. Our discussion of CP violation in the presence of new physics is aimed to demonstrate that, indeed, models of new physics can significantly modify the Standard Model predictions and that the near future measurements will therefore have a strong impact on the theoretical understanding of CP violation.

To understand how the Standard Model predictions could be modified by New Physics, we focus on CP violation in the interference between decays with and without mixing. As explained above, this type of CP violation may give, due to its theoretical cleanliness, unambiguous evidence for New Physics most easily. We now demonstrate what type of questions can be answered when many such observables are measured.

Consider $a_{\psi K_S}$, the CP asymmetry in $B \to \psi K_S$. This measurement will cleanly determine the relative phase between the $B - \bar{B}$ mixing amplitude and the $b \to c\bar{c}s$ decay amplitude ($\sin 2\beta$ in the SM). The $b \to c\bar{c}s$ decay has Standard Model tree contributions and therefore is very unlikely to be significantly affected by new physics. On the other hand, the mixing amplitude can be easily modified by new physics. We parametrize such a modification by a phase θ_d:

$$2\theta_d = \arg(M_{12}/M_{12}^{\rm SM}). \tag{7.1}$$

Then

$$a_{\psi K_S} = \sin[2(\beta + \theta_d)]. \tag{7.2}$$

Examining whether $a_{\psi K_S}$ fits the SM prediction, that is, whether $\theta_d \neq 0$, we can answer the following question (see *e.g.* [120]):

(i) *Is there new physics in $B-\bar{B}$ mixing*?

It is interesting to note that already now the measured value of $a_{\psi K_S}$ (1.3), which is consistent with the SM range, excludes many models that require a modification of CP violation in $B-\bar{B}$ mixing due to new physics. Among these are various models of soft CP violation [121,122] aimed to solve the strong CP problem, models of geometric CP violation due to extra dimensions [123], models of spontaneous CP violation in the left-right symmetric framework [124,125], and several models that aim to solve the supersymmetric CP problems [126–128].

Next consider $a_{\phi K_S}$, the CP asymmetry in $B \to \phi K_S$. This measurement will cleanly determine the relative phase between the $B-\bar{B}$ mixing amplitude and the $b \to s\bar{s}s$ decay amplitude ($\sin 2\beta$ in the SM). The $b \to s\bar{s}s$ decay has only Standard Model penguin contributions and therefore is sensitive to new physics. We parametrize the modification of the decay amplitude by a phase θ_A [129]:

$$\theta_A = \arg(\bar{A}_{\phi K_S}/\bar{A}^{\rm SM}_{\phi K_S}). \tag{7.3}$$

Then

$$a_{\phi K_S} = \sin[2(\beta + \theta_d + \theta_A)]. \tag{7.4}$$

Comparing $a_{\phi K_S}$ to $a_{\psi K_S}$, that is, examining whether $\theta_A \neq 0$, we can answer the following question:

(ii) *Is the new physics related to $\Delta B = 1$ processes? $\Delta B = 2$? both?*

Consider $a_{\pi\nu\bar{\nu}}$, the CP violating ratio of $K \to \pi\nu\bar{\nu}$ decays, defined in (4.22). This measurement will cleanly determine the relative phase between the $K-\bar{K}$ mixing amplitude and the $s \to d\nu\bar{\nu}$ decay amplitude (of order $\sin^2\beta$ in the SM). The experimentally measured small value of ε_K requires that the phase of the $K-\bar{K}$ mixing amplitude is not modified from the Standard Model prediction. (More precisely, it requires that the phase of the mixing amplitude is very close to twice the phase of the $s \to d\bar{u}u$ decay amplitude [130].) On the other hand, the decay, which in the SM is a loop process with small mixing angles, can be easily modified by new physics. Examining whether the SM correlation between $a_{\pi\nu\bar{\nu}}$ and $a_{\psi K_S}$ is fulfilled, we can answer the following question:

(iii) *Is the new physics related to the third generation? to all generations?*

Consider $a_{D\to K\pi}$, the CP violating quantity in $D \to K^\pm\pi^\mp$ decays defined in (5.10). It depends on ϕ_D, the relative phase between the $D-\bar{D}$ mixing amplitude and the $c \to d\bar{s}u$ and $c \to s\bar{d}u$ decay amplitudes. Within the Standard Model, the two decay channels are tree level. It is unlikely that they are affected by new physics. On the other hand, the mixing amplitude can be easily modified by new physics. Examining whether $a_{D\to K\pi} = 0$, that is, whether ϕ_D (and/or θ_d)$\neq 0$, we can answer the following question:

(iv) *Is the new physics related to the down sector? the up sector? both?*

Consider d_N, the electric dipole moment of the neutron. We did not discuss this quantity so far because, unlike CP violation in meson decays, flavor changing couplings are not necessary for d_N. In other words, the CP violation that induces d_N is *flavor diagonal*. It does in general get contributions from flavor changing physics, but it could be induced by sectors that are flavor blind. Within the SM (and ignoring $\theta_{\rm QCD}$), the contribution from $\delta_{\rm KM}$ arises at the three loop level and is at least six orders of magnitude below the experimental bound (1.7). If the bound is further improved (or a signal observed), we can answer the following question:

(v) *Are the new sources of CP violation flavor changing? flavor diagonal? both?*

It is no wonder then that with such rich information, flavor and CP violation provide an excellent probe of new physics. We will now demostrate this situation more concretely by discussing CP violation in supersymmetry.

B. The Supersymmetric Framework

Supersymmetry solves the fine-tuning problem of the Standard Model and has many other virtues. But at the same time, it leads to new problems: baryon number violation, lepton number violation, large flavor changing neutral current processes and large CP violation. The first two problems can be solved by imposing R-parity on supersymmetric models. There is no such simple, symmetry-related solution to the problems of flavor and CP violation. Instead, suppression of the relevant couplings can be achieved by demanding very constrained strcutures of the soft supersymmetry breaking terms. There are two important questions here: First, can theories of dynamical supersymmetry breaking naturally induce such structures? (For an excellent review of dynamical supersymmetry breaking, see [131].) Second, can measurements of flavor changing and/or CP violating processes shed light on the structure of the soft supersymmetry breaking terms? Since the answer to both questions is in the affirmative, we conclude that flavor changing neutral current processes and, in particular, CP violating observables will provide clues to the crucial question of how supersymmetry breaks.

1. CP Violating Parameters

A generic supersymmetric extension of the Standard Model contains a host of new flavor and CP violating parameters. (For a review of CP violation in supersymmetry see [132,133].) It is an amusing exercise to count the number of parameters [134]. The supersymmetric part of the Lagrangian depends, in addition to the three gauge couplings of $G_{\rm SM}$, on the parameters of the superpotential W:

$$W=\sum_{i,j}\left(Y^u_{ij}H_uQ_{Li}\overline{U}_{Lj}+Y^d_{ij}H_dQ_{Li}\overline{D}_{Lj}+Y^\ell_{ij}H_dL_{Li}\overline{E}_{Lj}\right)+\mu H_uH_d. \tag{7.5}$$

In addition, we have to add soft supersymmetry breaking terms:

$$\begin{aligned}\mathcal{L}_{\rm soft}=&-\left(A^u_{ij}H_u\tilde{Q}_{Li}\tilde{\overline{U}}_{Lj}+A^d_{ij}H_d\tilde{Q}_{Li}\tilde{\overline{D}}_{Lj}+A^\ell_{ij}H_d\tilde{L}_{Li}\tilde{\overline{E}}_{Lj}+BH_uH_d+{\rm h.c.}\right)\\&-\sum_{\rm all\ scalars}(m_S^2)_{ij}A_i\bar{A}_j-\frac{1}{2}\sum_{(a)=1}^{3}\left(\tilde{m}_{(a)}(\lambda\lambda)_{(a)}+{\rm h.c.}\right).\end{aligned} \tag{7.6}$$

where $S = Q_L, \overline{D}_L, \overline{U}_L, L_L, \overline{E}_L$. The three Yukawa matrices Y^f depend on 27 real and 27 imaginary parameters. Similarly, the three A^f-matrices depend on 27 real and 27 imaginary parameters. The five m_S^2 hermitian 3×3 mass-squared matrices for sfermions have 30 real parameters and 15 phases. The gauge and Higgs sectors depend on

$$\theta_{\rm QCD}, \tilde{m}_{(1)}, \tilde{m}_{(2)}, \tilde{m}_{(3)}, g_1, g_2, g_3, \mu, B, m_{h_u}^2, m_{h_d}^2, \tag{7.7}$$

that is 11 real and 5 imaginary parameters. Summing over all sectors, we get 95 real and 74 imaginary parameters. The various couplings (other than the gauge couplings) can be thought of as spurions that break a global symmetry,

$$U(3)^5 \times U(1)_{\rm PQ} \times U(1)_R \ \rightarrow \ U(1)_B \times U(1)_L. \tag{7.8}$$

The $U(1)_{\rm PQ} \times U(1)_R$ charge assignments are:

$$\begin{array}{cccccc} & H_u & H_d & Q\overline{U} & Q\overline{D} & L\overline{E} \\ U(1)_{\rm PQ} & 1 & 1 & -1 & -1 & -1 \\ U(1)_{\rm R} & 1 & 1 & 1 & 1 & 1 \end{array}. \tag{7.9}$$

Consequently, we can remove 15 real and 30 imaginary parameters, which leaves

$$124 \ = \ \begin{cases} 80 & \text{real} \\ 44 & \text{imaginary} \end{cases} \ \text{physical parameters.} \tag{7.10}$$

In particular, there are 43 new CP violating phases! In addition to the single Kobayashi-Maskawa of the SM, we can put 3 phases in M_1, M_2, μ (we used the $U(1)_{\rm PQ}$ and $U(1)_R$ to remove the phases from μB^* and M_3, respectively) and the other 40 phases appear in the mixing matrices of the fermion-sfermion-gaugino couplings. (Of the 80 real parameters, there are 11 absolute values of the parameters in (7.7), 9 fermion masses, 21 sfermion masses, 3 CKM angles and 36 SCKM angles.) Supersymmetry provides a nice example to our statement that reasonable extensions of the Standard Model may have more than one source of CP violation.

The requirement of consistency with experimental data provides strong constraints on many of these parameters. For this reason, the physics of flavor and CP violation has had a profound impact on supersymmetric model building. A discussion of CP violation in this context can hardly avoid addressing the flavor problem itself. Indeed, many of the supersymmetric models that we analyze below were originally aimed at solving flavor problems. For details on the supersymmetric flavor problem, see [135].

As concerns CP violation, one can distinguish two classes of experimental constraints. First, bounds on nuclear and atomic electric dipole moments determine what is usually called the *supersymmetric CP problem*. Second, the physics of neutral mesons and, most importantly, the small experimental value of ε_K pose the *supersymmetric ε_K problem*. In the next two subsections we describe the two problems.

2. The Supersymmetric CP Problem

One aspect of supersymmetric CP violation involves effects that are flavor preserving. Then, for simplicity, we describe this aspect in a supersymmetric model without additional flavor mixings, *i.e.* the minimal supersymmetric standard model (MSSM) with universal sfermion masses and with the trilinear SUSY-breaking scalar couplings proportional to the corresponding Yukawa couplings. (The generalization to the case of non-universal soft terms is straightforward.) In such a constrained framework, there are four new phases beyond the two phases of the SM ($\delta_{\rm KM}$ and $\theta_{\rm QCD}$). One arises in the bilinear μ-term of the superpotential (7.5), while the other three arise in the soft supersymmetry breaking parameters of (7.6): $\tilde{m}$ (the gaugino mass), A (the trilinear scalar coupling) and B (the bilinear scalar coupling). Only two combinations of the four phases are physical [136,137]:

$$\phi_A = \arg(A^* \tilde{m}), \quad \phi_B = \arg(\tilde{m}\mu B^*). \tag{7.11}$$

In the more general case of non-universal soft terms there is one independent phase ϕ_{A_i} for each quark and lepton flavor. Moreover, complex off-diagonal entries in the sfermion mass-squared matrices represent additional sources of CP violation.

The most significant effect of ϕ_A and ϕ_B is their contribution to electric dipole moments (EDMs). For example, the contribution from one-loop gluino diagrams to the down quark EDM is given by [138,139]:

$$d_d = m_d \frac{e\alpha_3}{18\pi\tilde{m}^3}\left(|A|\sin\phi_A + \tan\beta|\mu|\sin\phi_B\right), \tag{7.12}$$

where we have taken $m_Q^2 \sim m_D^2 \sim m_{\tilde{g}}^2 \sim \tilde{m}^2$, for left- and right-handed squark and gluino masses. We define, as usual, $\tan\beta = \langle H_u \rangle / \langle H_d \rangle$. Similar one-loop diagrams give rise to chromoelectric dipole moments. The electric and chromoelectric dipole moments of the light quarks (u, d, s) are the main source of d_N (the EDM of the neutron), giving [140]

$$d_N \sim 2\left(\frac{100\,GeV}{\tilde{m}}\right)^2 \sin\phi_{A,B} \times 10^{-23}\ e\,{\rm cm}, \tag{7.13}$$

where, as above, $\tilde{m}$ represents the overall SUSY scale. In a generic supersymmetric framework, we expect $\tilde{m} = \mathcal{O}(m_Z)$ and $\sin\phi_{A,B} = \mathcal{O}(1)$. Then the constraint (1.7) is generically violated by about two orders of magnitude. This is *the Supersymmetric CP Problem.*

Eq. (7.13) shows two possible ways to solve the supersymmetric CP problem:

(i) Heavy squarks: $\tilde{m} \gtrsim 1\ TeV$;

(ii) Approximate CP: $\sin\phi_{A,B} \ll 1$.

Recently, a third way has been investigated, that is cancellations between various contributions to the electric dipole moments. However, there seems to be no symmetry that can guarantee such a cancellation. This is in contrast to the other two mechanisms mentioned above that were shown to arise naturally in specific models. We therefore do not discuss any further this third mechanism.

The electric dipole moment of the electron is also a sensitive probe of flavor diagonal CP phases. The present experimental bound,

$$|d_e| \leq 4 \times 10^{-27}\ e\ {\rm cm}\ [141], \tag{7.14}$$

is also violated by about two orders of magnitue for 'natural' values of supersymmetric parameters. A new experiment [142] has been proposed to search for the electric dipole moment of the muon at a level smaller by five orders of magnitude than present bounds; such improvement will make d_μ another sensitive probe of supersymmetry [143].

The supersymmetric contribution to the ε_K parameter is dominated by diagrams involving Q and $\bar{d}$ squarks in the same loop. For $\tilde{m} = m_{\tilde{g}} \simeq m_Q \simeq m_D$ (our results depend only weakly on this assumption) and focusing on the contribution from the first two squark families, one gets (see, for example, [144]):

$$\varepsilon_K = \frac{5\ \alpha_3^2}{162\sqrt{2}} \frac{f_K^2 m_K}{\tilde{m}^2 \Delta m_K} \left[\left(\frac{m_K}{m_s + m_d} \right)^2 + \frac{3}{25} \right] \mathrm{Im} \left[(\delta_{12}^d)_{LL} (\delta_{12}^d)_{RR} \right]. \tag{7.15}$$

Here

$$(\delta_{12}^d)_{LL} = \left(\frac{m_{\tilde{Q}_2}^2 - m_{\tilde{Q}_1}^2}{m_{\tilde{Q}}^2} \right) \left| K_{12}^{dL} \right|,$$
$$(\delta_{12}^d)_{RR} = \left(\frac{m_{\tilde{D}_2}^2 - m_{\tilde{D}_1}^2}{m_{\tilde{D}}^2} \right) \left| K_{12}^{dR} \right|, \tag{7.16}$$

where K_{12}^{dL} (K_{12}^{dR}) are the mixing angles in the gluino couplings to left-handed (right-handed) down quarks and their scalar partners. Note that CP would be violated even if there were two families only [145]. Using the experimental value of ε_K, we get

$$\frac{(\Delta m_K \varepsilon_K)^{\mathrm{SUSY}}}{(\Delta m_K \varepsilon_K)^{\mathrm{EXP}}} \sim 10^7 \left(\frac{300\ GeV}{\tilde{m}} \right)^2 \left(\frac{m_{\tilde{Q}_2}^2 - m_{\tilde{Q}_1}^2}{m_{\tilde{Q}}^2} \right) \left(\frac{m_{\tilde{D}_2}^2 - m_{\tilde{D}_1}^2}{m_{\tilde{D}}^2} \right) |K_{12}^{dL} K_{12}^{dR}| \sin\phi, \tag{7.17}$$

where ϕ is the CP violating phase. In a generic supersymmetric framework, we expect $\tilde{m} = \mathcal{O}(m_Z)$, $\delta m_{Q,D}^2 / m_{Q,D}^2 = \mathcal{O}(1)$, $K_{ij}^{Q,D} = \mathcal{O}(1)$ and $\sin\phi = \mathcal{O}(1)$. Then the constraint (7.17) is generically violated by about seven orders of magnitude.

The Δm_K constraint on $\mathrm{Re}\left[(\delta_{12}^d)_{LL} (\delta_{12}^d)_{RR} \right]$ is about two orders of magnitude weaker. One can distinguish then three interesting regions for $\langle \delta_{12}^d \rangle = \sqrt{(\delta_{12}^d)_{LL} (\delta_{12}^d)_{RR}}$:

$$\begin{array}{rl} 0.003 \ll \langle \delta_{12}^d \rangle & \text{excluded}, \\ 0.0002 \ll \langle \delta_{12}^d \rangle \lesssim 0.003 & \text{viable with small phases}, \\ \langle \delta_{12}^d \rangle \ll 0.0002 & \text{viable with } \mathcal{O}(1) \text{ phases}. \end{array} \tag{7.18}$$

The first bound comes from the Δm_K constraint (assuming that the relevant phase is not particularly close to $\pi/2$). The bounds here apply to squark masses of order 500 GeV and scale like $\tilde{m}$. There is also dependence on $m_{\tilde{g}}/\tilde{m}$, which is here taken to be one.

Eq. (7.17) also shows what are the possible ways to solve the supersymmetric ε_K problem:
(i) Heavy squarks: $\tilde{m} \gg 300\ GeV$;
(ii) Universality: $\delta m_{Q,D}^2 \ll m_{Q,D}^2$;
(iii) Alignment: $|K_{12}^d| \ll 1$;
(iv) Approximate CP: $\sin\phi \ll 1$.

4. A Supersymmetric ε'/ε?

In this section we discuss the question of whether supersymmetric contributions to ε'/ε can be dominant. A typical supersymmetric contribution to ε'/ε is given by [146]

$$|\varepsilon'/\varepsilon| = 58 B_G \quad \left[\frac{\alpha_s(m_{\tilde g})}{\alpha_s(500\text{ GeV})}\right]^{23/21} \left(\frac{158\ MeV}{m_s + m_d}\right) \times \left(\frac{500\text{ GeV}}{m_{\tilde g}}\right) \left|\text{Im}\left[(\delta^d_{LR})_{12} - (\delta^d_{LR})^*_{21}\right]\right| , \tag{7.19}$$

where B_G parameterizes the matrix element of the relevant four-quark operator. Consequently, the supersymmetric contribution saturates ε'/ε for

$$\text{Im}\left[(\delta^d_{LR})_{12} - (\delta^d_{LR})^*_{21}\right] \ \sim\ \lambda^7 \left(\frac{m_{\tilde g}}{500\text{ GeV}}\right) \tag{7.20}$$

where, motivated by flavor symmetries (see below), we parameterize the suppression by powers of $\lambda \sim 0.2$.

Without proportionality, a naive guess would give

$$\begin{aligned}(\delta^d_{LR})_{12} &\sim \frac{m_s|V_{us}|}{\tilde m} \sim \lambda^{5-6}\frac{m_t}{\tilde m}, \\ (\delta^d_{LR})_{21} &\sim \frac{m_d}{|V_{us}|\tilde m} \sim \lambda^{5-6}\frac{m_t}{\tilde m} .\end{aligned} \tag{7.21}$$

This is not far from the value required to account for ε'/ε [147]. Thus, it is certainly *possible* that supersymmetry accounts for, at least, a large part of ε'/ε (see, for example, the models of refs. [148–154]). Yet, it has been argued that such a situation is not *generic* [155]. The problem is that eq. (7.21) gives an overestimate of the supersymmetric contribution in most viable models of supersymmetry breaking that have appeared in the literature. We will encounter concrete examples to this statement when we survey the various supersymmetric flavor models.

C. Supersymmetry Breaking and Flavor Models

Before turning to a detailed discussion, we define two scales that play an important role in supersymmetry: Λ_S, where the soft supersymmetry breaking terms are generated, and Λ_F, where flavor dynamics takes place. When $\Lambda_F \gg \Lambda_S$, it is possible that there are no genuinely new sources of flavor and CP violation. This leads to models with exact universality. When $\Lambda_F \lesssim \Lambda_S$, we do not expect, in general, that flavor and CP violation are limited to the Yukawa matrices. One way to suppress CP violation would be to assume that, similarly to the Standard Model, CP violating phases are large, but their effects are screened, possibly by the same physics that explains the various flavor puzzles, such as models with Abelian or non-Abelian horizontal symmetries. It is also possible that CP violating effects are suppressed because squarks are heavy. Another option, which is now excluded, was to assume that CP is an approximate symmetry of the full theory (namely, CP violating phases are all small).

If at some high energy scale squarks are exactly degenerate and the A terms proportional to the Yukawa couplings, then the contribution to ε_K comes from RGE and is GIM suppressed, that is

$$\varepsilon_K \propto \mathrm{Im}[(V_{td}V_{ts}^*)^2]Y_t^4 \left[\frac{\log(\Lambda_S/m_W)}{16\pi^2}\right]^2 . \tag{7.22}$$

This contribution is negligibly small [136]. The contribution from genuinely supersymmetric phases (*i.e.* the phases in A_t and μ) is also negligible [156,157]. (This does not necessarily mean that there is no supersymmetric effect on ε_K. In some small corner of parameter space the supersymmetric contribution from stop-chargino diagrams can give up to 20% of ε_K [158,159].)

In models of Gauge Mediated Supersymmetry Breaking (GMSB) [160,161], superpartner masses are generated by the SM gauge interactions. These masses are then exactly universal at the scale Λ_S at which they are generated (up to tiny high order effects associated with Yukawa couplings). Furthermore, A terms are suppressed by loop factors. The only contribution to ε_K is then from the running, and since Λ_S is low it is highly suppressed.

These models can also readily satisfy the EDM constraints. In most models, the A terms and gaugino masses arise from the same supersymmetry breaking auxiliary field, that is, they are generated by the same SUSY and $U(1)_R$ breaking source. They therefore carry the same phase (up to corrections from the Standard Model Yukawa couplings), and ϕ_A vanishes to a very good approximation:

$$\phi_A \propto Y_t^4 Y_c^2 Y_b^2 J_{\mathrm{CKM}} \left[\frac{\log(\Lambda_S/m_W)}{16\pi^2}\right]^4 . \tag{7.23}$$

The resulting EDM is $d_N \lesssim 10^{-31}$ e cm. This maximum can be reached only for very large $\tan\beta \sim 60$ while, for small $\tan\beta \sim 1$, d_N is about 5 orders of magnitude smaller. This range of values for d_N is much below the present ($\sim 10^{-25}$ e cm) and foreseen ($\sim 10^{-28}$ e cm) experimental sensitivities (see *e.g.* [162]).

The value of ϕ_B in general depends on the mechanism for generating the μ term. However, running effects can generate an adequate B term at low scales in these models even if $B(\Lambda_S) = 0$. One then finds [163]

$$B/\mu = A_t(\Lambda_S) + M_2(\Lambda_S)\left(-0.12 + 0.17|Y_t|^2\right) , \tag{7.24}$$

where M_2 is the $SU(2)$ gaugino mass. Since $\phi_A \simeq 0$, the resulting ϕ_B vanishes, again up to corrections involving the Standard Model Yukawa couplings [164].

There is therefore no CP problem in simple models of gauge mediation, even with phases of order one. The supersymmetric contribution to $D - \bar{D}$ mixing is similarly small and we expect no observable effects. As concerns the B_d system, GMSB models predict then a large CP asymmetry in $B \to \psi K_S$, with small deviations (at most 20%) from the SM.

More generally, in any supersymmetric model where there are no new flavor violating sources beyond the Yukawa couplings, CP violation in meson decays is hardly modified from the SM predictions [165].

If different moduli of string theory obtain supersymmetry breaking F terms, they would typically induce flavor-dependent soft terms through their tree-level couplings to Standard Model fields. There are however various scenarios in which the leading contribution to the soft terms is flavor independent. The three most intensively studied frameworks are dilaton dominance, anomaly madiation and gaugino mediation.

Dilaton dominance assumes that the dilaton F term is the dominant one. Then, at tree level, the resulting soft masses are universal and the A terms proportional to the Yukawa couplings. Both universality and proportionality are, however, violated by string loop effects. These induce corrections to squark masses of order $\frac{\alpha_X}{\pi} m_{3/2}^2$, where $\alpha_X = [2\pi(S+S^*)]^{-1}$ is the string coupling. There is no reason why these corrections would be flavor blind. However, RGE effects enhance the universal part of the squark masses by roughly a factor of five, leaving the off-diagonal entries essentially unchanged. The flavor suppression factor is then [166]

$$\langle \delta_{12}^d \rangle \simeq \frac{m_{12}^{2\ \mathrm{one-loop}}}{m_{\tilde{q}}^2} \simeq \frac{\alpha_X}{\pi} \frac{1}{25} \simeq 4 \times 10^{-4} \ . \tag{7.25}$$

Dilaton dominance relies on the assumption that loop corrections are small. This probably presents the most serious theoretical difficulty for this idea, because it is hard to see how non-perturbative effects, which are probably required to stabilize the dilaton, could do so in a region of weak coupling. In the strong coupling regime, these corrections could be much larger. However, this idea at least gives some plausible theoretical explanation for how universal masses might emerge in hidden sector models. Given that dilaton stabilization might require that non-perturbative effects are important, the estimate of flavor suppression (7.25) might well turn out to be an underestimate.

We now turn to the flavor diagonal phases that enter in various EDMs. The phase ϕ_A vanishes at tree-level, so that [166,167] $\phi_A = \mathcal{O}(\alpha_X/\pi)$. [The smallness of ϕ_A implies that there is a suppression of $\mathcal{O}(\alpha_X/\pi) \sim 10^{-2}$ compared to (7.21) and the supersymmetric contribution to ε'/ε is small.] However, ϕ_B is unsuppressed, even when μ, and through it B, are generated by Kahler potential effects through supersymmetry breaking, in which case $B = 2m_{3/2}^*\mu$ [168]. While the size of $m_{3/2}$ is determined from the requirement that the cosmological constant vanishes, its phase remains arbitrary, and in fact depends on the phase of the constant term that is added to the superpotential in order to cancel the cosmological constant.

We conclude that the supersymmetric ε_K problem is solved in these models but the EDM problem, in general, is not. For EDM contributions to be small in these models, the gravitino mass better give a small physical phase.

Anomaly mediation (AMSB) provides another approach to solving the flavor problems of supersymmetric theories, as well as to obtaining a predictive spectrum. In the presence of some truly 'hidden' supersymmetry breaking sector, with no couplings to the SM fields (apart from indirect couplings through the supergravity multiplet) the conformal anomaly of the Standard Model gives rise to soft supersymmetry breaking terms for the Standard Model fields [169,170]. These terms are generated purely by gravitational effects and are given by

$$m_0^2(\mu) = -\frac{1}{4}\frac{\partial\gamma(\mu)}{\ln\mu}m_{3/2}^2, \quad m_{1/2}(\mu) = \frac{\beta(\mu)}{g(\mu)}m_{3/2}, \quad A(\mu) = -\frac{1}{2}\gamma(\mu)m_{3/2} \ , \tag{7.26}$$

where β and γ are the appropriate beta function and anomalous dimension. Thus, apart from the Standard Model gauge and Yukawa couplings, the soft terms only involve the parameter $m_{3/2}$.

In general, naturalness considerations suggest that couplings of hidden and visible sectors should appear in the Kahler potential, leading to soft masses for scalars already at tree level, and certainly by one loop. As a result, one would expect the contributions (7.26) to be irrelevant. However, in "sequestered sector models" [169], in which the visible sector fields and supersymmetry breaking fields live on different branes separated by some distance, the anomaly mediated contribution (7.26) could be the dominant effect. This leads to a predictive picture for scalar masses. Since the soft terms (7.26) are generated by the Standard Model gauge and Yukawa couplings, they are universal, up to corrections involving the third generation Yukawa couplings. However, the resulting slepton masses-squared are negative, so this model requires some modification. We will not attempt a complete review of this subject here. Our principal concerns are the sources of CP violation, and the extent to which the AMSB formulae receive corrections, leading to non-degeneracy of the squark masses.

For eq. (7.26) to correctly give the leading order soft terms, it is necessary that all moduli obtain large masses before supersymmetry breaking, and that there be no Planck scale VEVs in the supersymmetry breaking sector [171]. A possible scenario for this to happen is if all moduli but the fifth dimensional radius, R, sit at an enhanced symmetry point, and that R obtains a large mass compared to the supersymmetry-breaking scale (say, by a racetrack mechanism). Even in this case, however, there is a difficulty. One might expect that some of the moduli have masses well below the fundamental scale. If there are light moduli in the bulk, there are typically one-loop contributions to scalar masses-squared from exchanges of bulk fields, proportional to $m_{3/2}^2/R^3$ times a loop factor [169]. If these contributions are non-universal, they may easily violate the Δm_K and ε_K constraints [133].

If there are no light moduli, and if the contributions described above are adequately suppressed, some modification of the visible sector is required in order to generate acceptable slepton masses. Different such solutions have been suggested. In some of these models, there are no new contributions to CP violation simply because there are few enough new parameters in the theory that they can all be chosen real by field redefinitions [172–174]. Furthermore, it is possible to generate the μ term in these models from AMSB, so that ϕ_B vanishes. These models are then similar to GMSB models from the point of view of CP violation.

We conclude then, that in generic sequestered sector models it is difficult to obtain strong degeneracy and a special phase structure is required. It is conceivable that there might be theories with a high degree of degeneracy, or with no new sources of CP violation. In such theories, the SM predictions for CP violation are approximately maintained.

Gaugino mediation [175,176] is in many ways similar to anomaly mediation, and poses similar issues. These models also suppress dangerous tree level contact terms by invoking extra dimensions, with the Standard Model matter fields localized on one brane and the supersymmetry breaking sector on another brane. In this case, however, the Standard

Model gauge fields are in the bulk, so gauginos get masses at tree level, and as a result scalar masses are generated by running. Scalar masses are therefore universal. Furthermore, the soft terms typically involve only one new parameter, namely, the singlet F VEV that gives rise to gaugino masses. Therefore, they do not induce any new CP violation.

Again, however, if there are non-universal tree and one loop contributions to scalar masses, significant violations of degeneracy and proportionality can be expected, and a special structure of CP violating phases is required.

3. Supersymmetric Flavor Models

Various frameworks have been suggested in which flavor symmetries, designed to explain the hierarchy of the Yukawa couplings, impose at the same time a special flavor structure on the soft supersymmetry breaking terms that helps to alleviate the flavor and CP problems.

In the framework of **alignment**, one does not assume any squark degeneracy. Instead, flavor violation is suppressed because the squark mass matrices are approximately diagonal in the quark mass basis. This is the case in models of Abelian flavor symmetries, in which the off-diagonal entries in both the quark mass matrices and in the squark mass matrices are suppressed by some power of a small parameter, λ, that quantifies the breaking of some Abelian flavor symmetry. A natural choice for the value of λ is $\sin\theta_C$, so we will take $\lambda \sim 0.2$. One would naively expect the first two generation squark mixing to be of the order of λ. However, the Δm_K constraint is not satisfied with the 'naive alignment', $K^d_{12} \sim \lambda$, and one has to construct more complicated models to achieve the required suppression [177,178]. One can solve the supersymmetric ε_K problem by flavor suppression, that is, models with $\langle\delta^d_{12}\rangle \sim \lambda^5$ [179]. These models are highly constrained and almost unique. It is simpler to construct models where $\langle\delta^d_{12}\rangle \sim \lambda^3$ but the CP violating phases are also suppressed [128]. Such models predict that $a_{\psi K_S} \ll 1$ and are therefore now excluded. (Models with $\langle\delta^d_{12}\rangle \sim \lambda^3$ could still be viable with phases of order one if the RGE contributions enhance squark degeneracy.)

As concerns flavor diagonal phases, the question is more model dependent. There is however a way to suppress these phases without assuming approximate CP [179]. The mechanism requires that CP is spontaneously broken by the same fields that break the flavor symmetry ("flavons"). It is based on the observation that a Yukawa coupling and the corresponding A term carry the same horizontal charge and therefore their dependence on the flavon fields is similar. In particular, if a single flavon dominates a certain coupling, the CP phase is the same for the Yukawa coupling and for the corresponding A term and the resulting ϕ_A vanishes. Similarly, if the μ term and the B term depend on one (and the same) flavon, ϕ_B is suppressed.

As concerns ε'/ε, the ε_K constraint requires that the relevant terms are suppressed by at least a factor of λ^2 compared to (7.21) [155] and the contribution is therefore small.

We conclude that one can construct models in which an Abelian horizontal symmetry solves both the ε_K and the d_N problems. These models are however not the generic ones in this framework.

Non-Abelian horizontal symmetries can induce approximate degeneracy between the first two squark generations, thus relaxing the flavor and CP problems [180]. A review of ε_K in this class of models can be found in [132]. Quite generically, the supersymmetric contributions to ε_K are too large and require small phases (see, for example, the models of ref. [181]). There are however specific models where the ε_K problem is solved without the need for small phases [182,183]. Furthermore, universal contributions from RGE running might further relax the problem.

As concerns flavor diagonal phases, it is difficult (though not entirely impossible) to avoid $\phi_A \gtrsim \lambda^2 \sim 0.04$ [132]. This, however, might be just enough to satisfy the d_N constraint.

With a horizontal U(2) symmetry, the two contributions to ε'/ε in (7.19) cancel each other. (More generally, this happens for a symmetric A matrix with $A_{11} = 0$ [184].)

We conclude that, similar to models of Abelian flavor symmetries, one can construct models of non-Abelian symmetries in which the symmetry solves both the ε_K and the d_N problems. These models are however not the generic ones in this framework.

Finally, one can construct models of **heavy first two generation squarks**. Here, the basic mechanism to suppress flavor changing processes is actually flavor diagonal: $m_{\tilde{q}_{1,2}} \sim$ 20 TeV. Naturalness does not allow higher masses, but this mass scale is not enough to satisfy even the Δm_K constraint [185], and one has to invoke alignment, $K^d_{12} \sim \lambda$. This is still not enough to satisfy the ε_K constraint of eq. (7.17), and a somewhat small phase is required.

Three more comments are in order: First, in this framework, gauginos are significantly lighter than the first two generation squarks, and so RGE cannot induce degeneracy. Second, the large mass of the squarks is enough to solve the EDM related problems, and so it is only the ε_K constraint that motivates a special phase structure. Finally, the contribution to ε'/ε is negligibly small. Instead of (7.21), a more likely estimate is [155] $(\delta^d_{LR})_{ij} \sim \frac{m_Z (M_d)_{ij}}{(10\ \mathrm{TeV})^2}$, which suppresses the relevant matrix elements by a factor of order 10^4.

D. (S)Conclusions

We would like to emphasize the following points:

(i) For supersymmetry to be established, a direct observation of supersymmetric particles is necessary. Once it is discovered, then measurements of CP violating observables will be a very sensitive probe of its flavor structure and, consequently, of the mechanism of dynamical supersymmetry breaking.

(ii) It seems possible to distinguish between models of exact universality and models with genuine supersymmetric flavor and CP violation. The former tend to give $d_N \lesssim 10^{-31}$ e cm while the latter usually predict $d_N \gtrsim 10^{-28}$ e cm.

(iii) The proximity of $a_{\psi K_S}$ to the SM predictions is obviously consistent with models of exact universality. It disfavors models of heavy squarks such as that of ref. [185]. Models of flavor symmetries allow deviations of order 20% (or smaller) from the SM predictions. To be convincingly signalled, an improvement in the theoretical calculations that lead to the SM predictions for $a_{\psi K_S}$ will be required [186].

(iv) Alternatively, the fact that $K \to \pi\nu\bar{\nu}$ decays are not affected by most supersymmetric flavor models [187–189] is an advantage here. The Standard Model correlation between $a_{\pi\nu\bar{\nu}}$ and $a_{\psi K_S}$ is a much cleaner test than a comparison of $a_{\psi K_S}$ to the CKM constraints.

(v) The neutral D system provides a stringent test of alignment. Observation of CP violation in the $D \to K\pi$ decays will make a convincing case for new physics.

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