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# Resonant Amplification of ν Oscillations in Matter and Solar-Neutrino Spectroscopy.

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**Summary.** — For small mixing angles $\theta$ the amplification of ν oscillations in matter has the resonance form (resonance in neutrino energy or matter density). In the Sun resonance effect results in nontrivial changing (suppression) of ν-flux for a wide range of neutrino parameters $\Delta m^2 = (3\cdot 10^{-4} \div 10^{-8})\,(\text{eV})^2$, $\sin^2 2\theta > 10^{-4}$.

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Matter modifies vacuum oscillations. Main relations describing this effect have been established (1) and some applications were considered (2-5). In particular, it was remarked that matter suppresses the oscillations with large length ($L_\nu \gg R_S$) inside the Sun (1) as well as inside cores of collapsing stars (2,4). Matter effects for underground oscillation experiments are also evaluated (5).

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(2) L. WOLFENSTEIN: *Phys. Rev. D*, **20**, 2634 (1979).
(3) V. BARGER, K. WHISNANT, S. PAKVASA and R. J. N. PHILLIPS: *Phys. Rev. D*, **22**, 2718 (1980).
(4) S. PAKVASA: DUMAND-80, Vol. **2** (1981), p. 45; H. J. HAUBOLD: *Astrophys. Space Sci.*, **82**, 457 (1982).
(5) P. V. RAMANA MURTHY: *Proceedings of the XVIII International Cosmic Rays Conference*, Vol. **7** (Bangalore, 1983), p. 125.

In this paper the amplification of ν oscillations is considered. It will be stressed that at small $\sin^2 2\theta$ the amplification has the resonance character. Applications of this effect to the neutrino fluxes inside the Sun and in the dense stars are discussed.

## 1. – Resonant amplification of ν oscillations in matter.

Consider the mixing of two neutrinos: $\nu_e = c\cdot\nu_1 + s\cdot\nu_2$ and $\nu_\alpha = -s\cdot\nu_1 + c\cdot\nu_2$, where $\nu_\alpha = \nu_\mu$ or $\nu_\tau$ (« flavour » oscillations), or $\nu_\alpha = \bar{\nu}_e$ ($\nu_e$-$\bar{\nu}_e$ oscillations); $\nu_1$ and $\nu_2$ are the states with definite masses $m_1$ and $m_2$; $c = \cos\theta$, $s = \sin\theta$.

The matter modifies vacuum oscillations if the amplitudes of $\nu_e$ and $\nu_\alpha$ elastic forward scattering on electrons or/and nucleons are different [1]: $\Delta f(0) = f_e(0) - f_\alpha(0) \neq 0$ (*). Effect is twofold. 1) The difference in $\nu_e$ and $\nu_\alpha$ interactions implies that the $\nu_1$-$\nu_2$ transitions exist. This in its turn means that the eigenstates of propagation in matter $\nu_1^m$ and $\nu_2^m$ do not coincide with $\nu_1$ and $\nu_2$. Because the oscillation angle $\theta_m$ is determined with respect to $\nu_1^m$ and $\nu_2^m$ it differs from vacuum mixing angle $\theta$. 2) In matter the refraction index is not equal to unity and this results in the changing of the oscillation length. Corresponding relations are as follows [1]:

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(1 - 2\cdot L_\nu/L_0\cdot\cos 2\theta + L_m^2/L_0^2)}\,, \tag{1}$$

$$L_m = \frac{L_\nu}{(1 - 2\cdot L_\nu/L_0\cdot\cos 2\theta + L_\nu^2/L_0^2)^{\frac{1}{2}}}\,. \tag{2}$$

Here $L_\nu = 4\pi\cdot E/\Delta m^2$ is the oscillations length in vacuum; $l_0 = 2\pi/(\Delta f(0)/k\cdot\varrho\cdot Y/m_N)$ can be considered as the oscillations « eigenlength » in matter, $\varrho$ is the matter density, $m_N$ is nucleon mass, $Y$ is the number of particles per nucleon interacting with neutrinos. For definiteness we will suppose that $\theta < 45°$ and that $\Delta m^2$ can be both positive and negative.

Our main point is that the dependence of oscillations depth ($\sin^2 2\theta_m$) on $L_\nu/L_0$ (or equivalently on $E$ and $\varrho$) has the resonance character (fig. 1). Indeed, at small $L_\nu/L_0$ matter effects are inessential: $\sin^2 2\theta_m \simeq \sin^2 2\theta$. When $L_\nu/L_0$

(*) In the case of « flavour » oscillations ($\nu_e$-$\nu_\mu$ or $\nu_\tau$) nonvanishing $\Delta f(0)$ results from $\nu_e$-scattering on electrons due to charged current: $\Delta f(0) = \sqrt{2}\,Gk$ [3]. Because $\nu_{e_L}$ does not participate in the weak interaction for $\nu_{e_L}$-$\nu_{e_L}$ oscillations both charged and neutral currents on electrons as well as neutral current on nucleons give contributions in $\Delta f(0)$. In the electrically and isotopically neutral medium $f(0)$ is two times less than for $\nu_e$-$\nu_\mu$ transitions: $\Delta f(0) = \sqrt{2}\,Gk/2$. In the electrically neutral medium without neutrons $\Delta f(0)$ coincides with one for $\nu_e$-$\nu_\mu$. Further results correspond to $\nu_e$-$\nu_\mu$, $\nu_\tau$ case.

approaches cos $2\theta$ the depth of oscillations increases and at

(3) $$\frac{L_\nu}{L_0} = \cos 2\theta$$

it achieves the maximal value $\sin^2 2\theta_m = 1$. Beyond the maximum as well as at negative value $L_\nu/L_0$ matter suppresses oscillations. In particular, for $|L_\nu/L_0| \gg 1$ one has $\sin^2 2\theta_m \simeq \sin^2 2\theta/(L_\nu/L_0)^2$. The width of resonance curve is

(4) $$\Delta\left(\frac{L_\nu}{L_0}\right) = \left(\frac{L_\nu}{L_0}\right)_{res} \cdot \text{tg} 2\theta = \sin 2\theta \, .$$

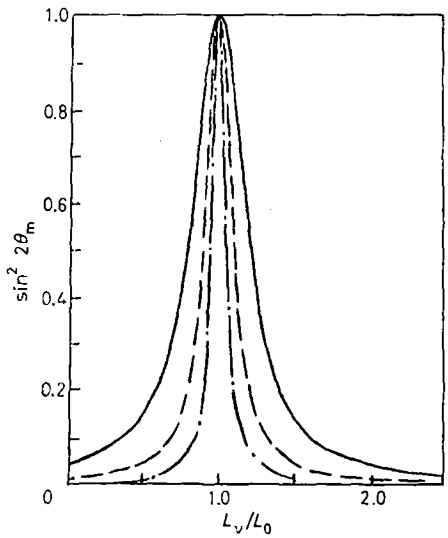


Fig. 1. – The dependence of oscillation depth $\sin^2 2\theta_m$ on $L_\nu/L_0 = (E/m^2)\cdot(f(0)/k)\cdot(Y\varrho/m_N)$, $\sin^2 2\theta$: —— $4\cdot 10^{-2}$, – – – $10^{-2}$, —·—·— $2.5\cdot 10^{-3}$.

Thus the smaller $\theta$ is, the narrower is the resonant peak. As $\theta$ diminishes, the value $L_\nu/L_0$ corresponding to the maximum approaches the unity.

The sense of such effect is quite simple. When the eigenfrequency of the system (in our case $L_\nu$) coincides with the eigenfrequency of external medium (here $L_0 \cdot \cos 2\theta \approx L_0$ at small $\theta$) the resonant amplification of oscillations should take place.

At fixed $L_\nu$ the matter oscillation length $L_m$ is maximal in resonance

(5) $$L_m = \frac{L_\nu}{|\sin 2\theta|} \, .$$

The resonance condition (3) can be rewritten as

(6) $$\frac{E}{\Delta m^2}\varrho = a \cdot \cos 2\theta \, ;$$

where $a = 0.5\cdot(m_{\mathcal{N}}/Y)\cdot(k/\Delta f(0))$. It follows from (6) that $\Delta f(0)$ and $\Delta m^2$ should have the same signs. In a given medium the resonant condition is fulfilled for $\nu$ or $\bar{\nu}$ only because $\Delta f(0)$ changes sign when $\nu$ is replaced by $\bar{\nu}$.

According to (6) one can suppose two realizations of resonant condition. 1) Neutrinos with continuous energy spectrum pass through the medium with constant density. In this case the oscillations will be amplified for neutrinos with energy in definite interval near $E_{\text{res}} = a\cdot\cos 2\theta\cdot\Delta m^2/\varrho$. 2) Neutrinos with definite energy propagate in the medium with varying density. Resonant amplification takes place in the layer with $\varrho_{\text{res}} - \Delta\varrho < \varrho < \varrho_{\text{res}} + \Delta\varrho$ where, according to (4) and (6)

$$\varrho_{\text{res}} = \frac{a\cdot\cos 2\theta}{E/\Delta m^2} \tag{7}$$

and

$$\Delta\varrho = \varrho_{\text{res}}\cdot\text{tg}2\theta = \frac{a\cdot\sin 2\theta}{E/\Delta m^2}\,. \tag{8}$$

We shall refer to such a layer as to a resonance one. In order to have deep oscillations to develop, the width of the resonance layer should be of order or more than the oscillation length in matter:

$$R_{\text{res}} > L_{\text{m}} = \frac{L_0}{\text{tg}2\theta}\,. \tag{9}$$

When conditions (7), (9) are fulfilled a large oscillation effect will take place even if the vacuum mixing is very small.

## 2. – Oscillations in matter with varying density.

From evolution equations for wave function $\nu_e(t)$, $\nu_\mu(t)$ (1) one can obtain the system of equations immediately for probabilities:

$$\begin{cases} (\mathrm{d}P/\mathrm{d}t) = -2\cdot\bar{M}\cdot I\,, \\ (\mathrm{d}I/\mathrm{d}t) = -M(t)\cdot C + \bar{M}(2\cdot P - 1)\,, \\ (\mathrm{d}C/\mathrm{d}t) = M(t)\cdot I\,, \end{cases} \tag{10}$$

where $P(t) = \langle\nu_e|\nu_e\rangle$ is the probability to find $\nu_e$ at the moment $t$; $C$ and $I$ are real and imaginary parts of matrix element $\langle\nu_\mu|\nu_e\rangle = C + iI$ correspondingly; $\bar{M} = \pi\sin 2\theta/L_\nu$, $\bar{M}$ is a parameter depending on the time: $M(t) = 2\pi(\cos 2\theta/L_\nu - 1/L_0(t))$, $L_0(t) = L_0\cdot\varrho(t)$ (see above). If the initial state is the electron neutrino one, then

$$P(0) = 1\,,\qquad I(0) = C(0) = 0\,. \tag{11}$$

Let the matter density decrease monotonically from some $\varrho_{max}$ to $\varrho_{min} = 0$. Three possibilities appear.

*A*) $\varrho_{res}$ (see (7)) is in the interval: $\varrho_{max} - \varrho_{min}$. The result of passing through the matter depends on the velocity of density changing: if

$$R_{res} = (d\varrho/dr)^{-1}\cdot\Delta\varrho = (d\varrho/dr)\frac{a\cdot\sin 2\theta}{E/\Delta m^2} > L_m/2\,, \tag{12}$$

then at small mixing angle $\theta$, $\nu_e$ turns into $\nu_\mu$ almost completely: the neutrino beam leaves the layer with small oscillation near $\langle P\rangle = \sin^2\theta$. Strong changing in ν-beam occurs in spite of « adiabatic » condition (12) and the fact that $\nu_e$ practically coincides with eigenstates in matter $(\nu_1^m, \nu_2^m)$ both at $t = 0$ and $t = \infty$. The reason is that the initial and final conditions of our system are in fact different. In the initial state $\varrho(0) = \varrho_{max}$ and $\nu_e$ coincides with $\nu_2^m(0)$, whereas in the final state $(\varrho = 0)$ $\nu_e$ is near $\nu_1^m(0)$. In other words, at $t = 0$ the mixing angle in matter $\theta$ is 90° (with respect to $\nu_1^m$) but at $t = \infty$ one has $\theta = 0°$. Note that, if $\varrho(0) = \varrho(\infty) = \varrho_{max}$ (or $\varrho_{min}$) then, in spite of the deep oscillations in the layers with $\varrho(t) \sim \varrho_{res}$, the initial and final neutrino states due to adiabatic condition (12) will be the same.

*B*) If condition (12) is not fulfilled ($R_{res} < L_m/2$—fast changing of density) there is no time for deep oscillation to develop. With decreasing $R_{res}/L_m$, $\langle P\rangle$ goes to the unit.

*C*) $\varrho_{res} > \varrho_{max}$. The resonant condition is not fulfilled in the layer. With $\varrho_{res}/\varrho_{max}$ increasing, $\langle P\rangle$ rises too.

Combining this three cases (*A*), *B*), *C*)) we can get the interpretation of result for arbitrary density distribution.

## 3. – ν oscillations in the Sun.

Consider now real objects in which resonant amplification of oscillations take place. We have found that the resonance conditions may be fulfilled in the Sun and in the cores as well as in the envelopes of massive collapsing stars. In the Earth the condition (9) for small $\theta$ is not fulfilled: $L_0/\text{tg}\,2\theta \gg L_0 = 1.77\cdot10^7$ m, whereas $R_{res} < R_E = 1.3\cdot10^7$ m. In this paper we concentrate on resonant amplification in the Sun.

The solar-neutrino problem is widely known [6,7]. Neutrino oscillations are

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(7) V. A. Kuzmin: *Phys. Lett.*, **17**, 27 (1965); I. R. Barabanov, A. I. Egorov, V. N. Gavrin, Yu. S. Kopysov and G. T. Zatsepin: *Neutrino-77*, Vol. **1** (Nauka, Moscow, 1978), p. 20; V. A. Kuzmin and G. T. Zatsepin: *Proceedings of the IX International Cosmic Rays Conference*, Vol. **2** (London, 1965), p. 1023.

considered as one of the possible reasons (8) of discrepancy between predicted (9) solar-neutrino flux and measured one in Cl-Ar experiment (6). Oscillations in vacuum, that is in the way from the Sun to the Earth ($L_E$), are discussed in detail (10). Appreciable suppression appears in the case of large mixing only.

The effect of matter on a long-range oscillations ($L_\nu > R_S$) was discussed (1,11). Inside the Sun one has $L_\nu/L_0 \gg 1$ and oscillations are suppressed. In this paper we deal with the region $L_\nu < R_S$ where the inverse effect—resonant amplification of oscillations—may take place.

Qualitatively the effect is as follows. For neutrinos with given energy $E$ (from sufficiently large interval of $E/\Delta m^2$) the layer exists inside the Sun where the resonance conditions (7), (9) are fulfilled. Neutrino flux generated under this layer (or inside it) and crossing it undergoes deep oscillations. The position of resonance layer, its width and, consequently, the value of the oscillation effect depend on $E/\Delta m^2$. Therefore, matter gives nontrivial suppression of neutrino spectrum.

Let us find the region of parameters $\Delta m^2$ and $\sin^2 2\theta$ for which the resonant amplification takes place. According to (6) the maximal (central) density of the Sun determines the lower border of the $E/\Delta m^2$ region:

$$\frac{E}{\Delta m^2} > \frac{a\cdot\cos 2\theta}{\varrho_{\max}} = 4\cdot 10^4\cdot\cos 2\theta\,. \tag{13}$$

Here $E$ is in MeV and $\Delta m^2$ is in $(\text{eV})^2$. For neutrinos with $E/\Delta m^2 = 4\cdot 10^4$ the resonant amplification occurs in the centre of the Sun. With increasing of $E/\Delta m^2$ the resonance layer is shifted to the surface of the Sun. Maximal energy in neutrino spectrum $E = 14$ MeV and restriction (10) give the upper limit on $\Delta m^2$: $\Delta m^2 < E_{\max}/4\cdot 10^4\cdot\cos 2\theta = 3\cdot 10^{-4}\,(\text{eV})^2$.

Maximal values of $E/\Delta m^2$ correspond to the resonance in surface layer of the Sun, where the density is small. The border of $\Delta m^2$, $\sin^2 2\theta$ region is determined by condition (9). At small $\varrho$ and small $\sin 2\theta$ the oscillation length becomes larger than the width of resonance layer and thus the deep oscillations do not develop. The real calculation of $(E/\Delta m^2)_{\max}$ and the minimal detectable energy of neutrinos 0.234 MeV (the threshold energy in Ga-Ge experiment) give the lower border on the resonance region $\Delta m^2$, $\sin^2 2\theta$ (see fig. 2).

Let us find the suppression factor $P(E, \Delta m^2, \sin^2 2\theta)$ for neutrino flux near the Earth as the function of $E$, $\Delta m^2$ and $\sin^2 2\theta$. In fig. 3 the dashed curves $P(E/\Delta m^2)$ corresponds to the pointlike source of neutrinos in the centre of the

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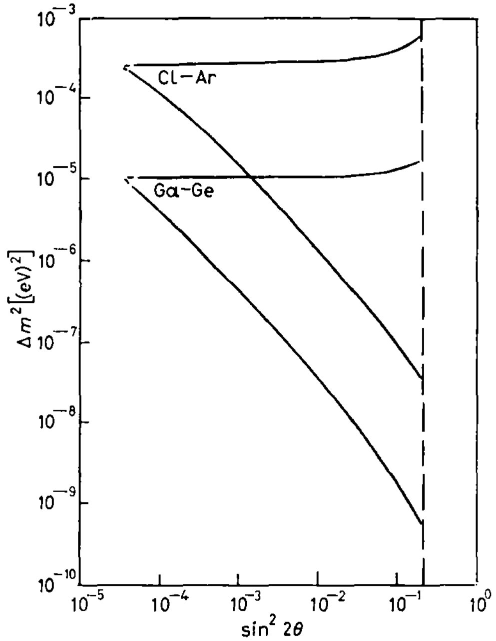


Fig. 2. – The region of neutrino parameters $m^2$, $\sin^2 2\theta$ for which resonant amplification of oscillations takes place in the Sun. Inside the regions limited by full lines the suppression factor for Cl-Ar and/or Ga-Ge experiments exceed 10%. The dashed line restrict the region of 10% effect due to vacuum oscillations only.

Sun. The system of equations (10) has been solved with the density distribution of standard solar model (9). For the point $E/\Delta m^2 = 4\cdot 10^4$ the resonance occurs in the centre of the Sun. In the region $E/\Delta m^2 < 4\cdot 10^4$ there is no resonance at all (case $C$)). In the wide region $E/\Delta m^2 < (E/\Delta m^2)_{max} = f(\sin^2 2\theta)$ condition (12) is fulfilled. So at $4\cdot 10^{-4} < E/\Delta m^2 < (E/\Delta m^2)_{max}$ case $A$) takes place. For $E/\Delta m^2 > (E/\Delta m^2)_{max}$ the resonant layer is thin: $R_{res} < L_m/2$ (case $B$)).

Solid curves in fig. 3 represent the probabilities $P(E/\Delta m^2)$ for neutrino production region with $R_{pr} = 0.2\cdot R_S$. The solid and dashed curves differ appreciably in the region $E/\Delta m^2 < 4\cdot 10^{-4}$. The reason is that the part of neutrinos with $E/\Delta m^2 > 4\cdot 10^{-4}$ produced in the forward hemisphere (with respect to Earth) does not pass through the resonant layer.

Consider now possible changing in solar-neutrino spectrum and in experimental data due to resonant oscillation. The solar-neutrino flux near the Earth is

$$F_\nu(E) = P(E, \Delta m^2, \sin^2 2\theta)\cdot F_\nu^0(E)\,,$$

where $F_\nu^0(E)$ is the ν-flux without oscillations. Depending on $\Delta m^2$ different parts of calculated curves will work. Obviously with the decrease of $\Delta m^2$

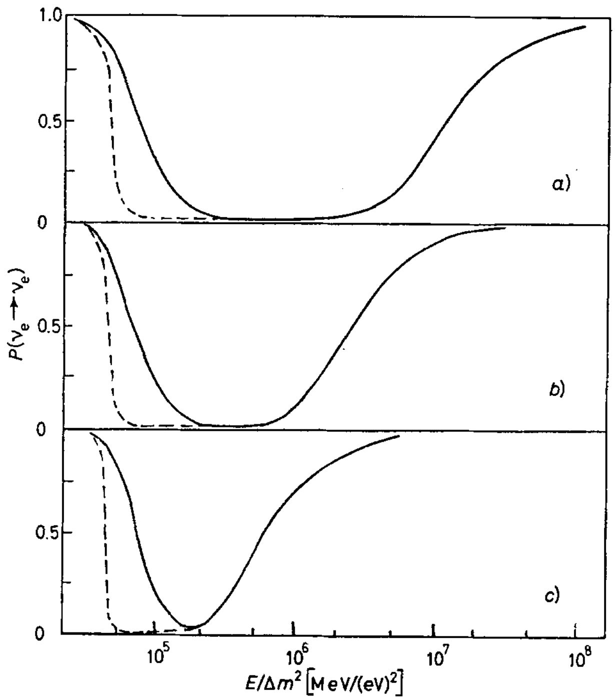


Fig. 3. – The suppression factor $P(E/m^2, \sin^2 2\theta)$ for neutrino production region with $R_{pr} = 0.2 \cdot R_S$ (full line) and for pointlike neutrino source in the centre of the Sun (dashed line). *a*) $\sin^2 2\theta = 4 \cdot 10^{-2}$, *b*) $\sin^2 2\theta = 10^{-2}$, *c*) $\sin^2 2\theta = 2.5 \cdot 10^{-3}$.

the neutrino spectrum is shifted in the region of large values $E/\Delta m^2$. At $10^{-5}\,(\text{eV})^2 < \Delta m^2 < 3 \cdot 10^{-4}\,(\text{eV})^2$ only $^8$B-neutrinos fall in the resonant region (see fig. 2). In the interval $10^{-9}\,(\text{eV})^2/\sin^2 2\theta < \Delta m^2 < 10^{-5}\,(\text{eV})^2$ all solar neutrinos undergo resonant amplification of oscillation. At $\Delta m^2 < 10^{-9}\,(\text{eV})^2/\sin^2 2\theta$ the resonance layer exists only for pp neutrinos.

## 4. – Discussion and results.

Now some remarks on the matter effect for 3 neutrino mixing. In the most natural case with the mass hierarchy ($m_1 \ll m_2 \ll m_3$) the results coincide with the considered ones or can be obtained from them in a simple way. More interesting situation is when at least two masses fall inside the resonant interval: $\Delta m^2_{max} > m_2^2, m_3^2 > \Delta m^2_{min}$. In this case the curve $P(E)$ will be the superposition of two considered ones with $\Delta m_1^2 = m_2^2$ and $\Delta m_2^2 = m_3^2$ and with the corresponding mixing angles.

One should remark that the resonance conditions can be fulfilled in the cores as well as in envelopes of collapsing stars. In contrast with the existing opinion that the dense matter suppresses oscillations [2,4], one can find (due to considered resonance effect) the region of oscillation parameters for which both oscillations take place with the maximal depth and the experimental constraints on oscillations are fulfilled. Indeed, for $\varrho = 10^9$ g/cm$^3$ and $\varrho = 10^{15}$ g/cm$^3$ one has from (6) $\Delta m^2 > 10^3$ (eV)$^2$ and $\Delta m^2 > 10^6$ (eV)$^2$ correspondingly. Reactors and accelerator restrictions can be fulfilled by small vacuum mixing angle ($\sin^2 2\theta < 10^{-2} \div 10^{-3}$). In that time due to resonant amplification one has $\sin^2 2\theta_m = 1$.

In conclusion (*):

1) Matter may amplify the depth of oscillations. At small $\sin^2 2\theta$ this amplification has a resonance character in $E$ and/or in $\varrho$.

2) The condition for the resonant amplification are fulfilled in the Sun as well as in the cores and envelopes of collapsing stars.

3) In the Sun the resonant oscillations results in the strong modification of neutrino spectrum for a wide region of $\Delta m^2$ and $\sin^2 2\theta$. Appreciable oscillation effect exists even for very small values of $\sin^2 2\theta$ ($10^{-4} \div 10^{-5}$).

4) The spectroscopy of all solar neutrinos gives information on the wide region of oscillation parameters:

$$\Delta m^2 = 3 \cdot 10^{-4} \div 10^{-8}\,(\text{eV})^2\,, \qquad \sin^2 2\theta > 10^{-4}\,.$$

* * *

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(*) We would like to emphasize once more that we deal with the modification by matter or « usual » vacuum oscillations which are due to the mass difference of neutrinos.

## ● RIASSUNTO (*)

Per piccoli angoli di mescolanemto $\theta$ l'amplificazione delle oscillazioni neutriniche nella materia ha la forma di risonanza (risonanza nell'energia neutrinica o densità di materia). Nel sole l'effetto di risonanza produce un cambiamento non banale (soppressione) del flusso di neutrini per un'ampia gamma di parametri neutrinici $\Delta m^2 = 3 \cdot 10^{-4} \div 10^{-8}$ (eV)$^2$, $\sin^2 2\theta > 10^{-4}$.

(*) *Traduzione a cura della Redazione.*

**Резонансное усиление ν осцилляций в веществе и спектроскопия солнечных нейтрино.**

**Резюме.** — Для малых углов смешивания усиление ν осцилляций в веществе имеет резонансную форму (резонанс по энергии нейтрино или плотности вещества). В Солнце резонансный эффект приводит к сложному изменению (подавлению) ν-потока для широкого диапазона нейтринных параметров: $\Delta m^2 = 3\cdot(10^{-4} \div 10^{-8})$ эВ$^2$, $\sin^2 2\theta > 10^{-4}$.