MEASUREMENT OF THE PHOTON STRUCTURE FUNCTION $F_2^V(x, Q^2)$

PLUTO Collaboration

Ch. BERGER, A. DEUTER, H. GENZEL, W. LACKAS, J. PIELORZ ¹, F. RAUPACH ², W. WAGNER ³
I. Physikalisches Institut der RWTH Aachen ⁴, Fed. Rep. Germany

A. KLOVNING, E. LILLESTØL
University of Bergen ⁵, Norway

J. BÜRGER ⁶, L. CRIEGER, F. FERRAROTTO ⁷, G. FRANCO, M. GASPERO ⁷, Ch. GERKE ⁸,
G. KNIES, B. LEWENDEL, J. MEYER, U. MICHELS, K.H. PAPE, B. STELLA ⁷, U. TIMM,
G.G. WINTER, M. ZACHARA ⁹, W. ZIMMERMANN
Deutsches Elektronen-Synchrotron (DESY), Hamburg, Fed. Rep. Germany

P.J. BUSSEY, S.L. CARTWRIGHT ¹⁰, J.B. DAINTON, B.T. KING, C. Raine, J.M. SCARR,
I.O. SKILLICORN, K.M. SMITH, J.C. THOMSON
University of Glasgow ¹¹, UK

O. ACHTERBERG ¹², V. BLOBEL, D. BURKART, K. DIEHL, M. FEINDT, H. KAPITZA,
B. KOPPITZ, M. KRÜGER ¹², M. POPPE, H. SPIETZER, R. van STAA
II. Institut für Experimentalphysik der Universität Hamburg ⁴, Fed. Rep. Germany

C.Y. CHANG, R.G. GLASSER, R.G. KELLOGG, S.J. MAXFIELD, R.O. POLVADO ¹³,
University of Maryland ¹⁴, USA

F. ALMEIDA ¹⁵, A. BÄCKER, F. BARREIRO, S. BRANDT, K. DERIKUM ¹⁶, C. GRUPEN,
H.J. MEYER, H. MÜLLER, B. NEUMANN, M. ROST, K. STUPPERICH, G. ZECH
Universität-Gesamthochschule Siegen ⁴, Fed. Rep. Germany

G. ALEXANDER, G. BELLA, Y. GNAT, J. GRUNHAUS
Tel-Aviv University ¹⁷, Israel

H. JUNGE, K. KRASKI, C. MAXEINER, H. MAXEINER, H. MEYER and D. SCHMIDT
Universität-Gesamthochschule Wuppertal ⁴, Fed. Rep. Germany

¹ Deceased.
² Present address: University of Paris Sud, Orsay, France.
³ Present address: University of California at Davis, CA, USA.
⁴ Supported by the BMFT, Federal Republic of Germany.
⁵ Partially supported by the Norwegian Council for Science and Humanities.
⁶ Currently on leave of absence at CERN, Geneva, Switzerland.
⁷ Rome University, partially supported by INFN, Sezione di Roma, Italy.
⁸ Present address: CERN, Geneva, Switzerland.
⁹ Institute of Nuclear Physics, Cracow, Poland.
¹⁰ Present address: Rutherford Appleton Laboratory, Chilton, England.
¹¹ Supported by the UK Science and Engineering Research Council.
¹³ Present address: Northeastern University, Boston, MA, USA.
¹⁴ Partially supported by the Department of Energy, USA.
¹⁵ On leave of absence from Instituto de Fisica, Universidad Federal do Rio de Janeiro, Brazil.
¹⁷ Partially supported by the Israeli Academy of Sciences and Humanities – Basic Research Foundation.
The structure function \( F_2^r \) for a quasi-real photon has been measured in the \( Q^2 \) range 1.5 to 16 GeV\(^2\) using 1417 multi-hadron events obtained with the PLUTO detector at PETRA. The \( x \) dependence of \( F_2^r \) has been corrected for the effects of experimental resolution and incomplete acceptance. The result is compared with theoretical expectations. With weak theoretical assumptions, bounds of \( 65 < \Lambda_{\overline{MS}} < 575 \) MeV are obtained for the QCD scale parameter.

The structure of the photon can be studied in deep inelastic \( e^+ e^- \) scattering at \( e^+ e^- \) storage rings. If either the electron or positron is scattered at a large angle \( \Theta_1 \) and the other one restricted to small angles \( \Theta_2 \ll \Theta_2^{\text{max}} \ll 1 \) rad, the highly virtual photon emitted in the large-angle scatter probes the structure of the other quasi-real photon. In this asymmetric configuration the cross section for the reaction

\[
e e^+ \rightarrow e^+ e^- + \text{hadrons},
\]

(1)

can be expressed in terms of the two structure functions \( F_2^r(x, Q^2) \) and \( F_1(x, Q^2) \) of the quasi-real photon as

\[
\frac{d\sigma}{dx} dQ^2 = (4\pi\alpha^2/Q^4)(1/x) \times \left[(1-y)F_2^r(x, Q^2) + xy^2 F_1(x, Q^2)\right] \times N_\gamma(z, \Theta_2^{\text{max}}) dz,
\]

(2)

where \( -Q^2 \) is the squared mass of the probing photon, \( x = Q^2/(Q^2 + W^2) \), \( y = 1 - (E'/E) \cos^2(\Theta_1/2) \), (3) are the scaling variables, \( W \) is the invariant mass of the produced hadron system, and \( E' \) is the energy of the electron at \( \Theta_1 \) which “tags” the probing photon \( E = E_{\text{beam}} \). The function

\[
N_\gamma(z, \Theta_2^{\text{max}}) = \left(\frac{\alpha}{\pi}\right)(1/z) \times \left[(1 + (1 - z)^2) \ln [E(1 - z)\Theta_2^{\text{max}}/m_c z] - (1 - z)\right],
\]

(4)

describes the energy spectrum of a beam of almost real “target” photons with squared mass \( -p^2 \) close to zero and fractional energy \( z = E_{\gamma}/E \). The factorisation of the \( ee \) cross section into an \( e^+ e^- \) cross section and a target photon flux holds not only for small scattering angles [1], but also for arbitrary \( Q^2 \), provided \( \Theta_2^{\text{max}} \ll 1 \) and \( W^2 \gg P^2 \) [2,3]. Because of the experimental cuts imposed on \( E' \) and \( \Theta_1 \), \( y \) is small (\( \langle y \rangle = 0.15 \)) so that the contribution from \( F_1^r \) in eq. (2) is negligible, and the measurement yields \( F_2^r \) directly.

The hadronic structure function of a photon \( F_2^r \) is of interest for the following reasons [4]:

(1) It is expected to contain a point-like contribution from the direct \( \gamma \gamma \rightarrow q\bar{q} \) coupling [the Born diagram of the quark parton model (QPM)] which can be calculated in perturbative QCD [5] and which is expected to dominate at large \( Q^2 \). In leading order this point-like contribution factorises

\[
F_2^{\text{LO}}(x, Q^2) = \frac{3\alpha}{\pi} \sum q_i^4 f_2^{\text{LO}}(x) \ln(Q^2/\Lambda^2),
\]

(5)

with \( f_2^{\text{LO}}(x) \) a rising function of the scaling variable \( x \). Thus in leading order QCD (as well as in the QPM) \( F_2^r \) increases with both \( x \) and \( Q^2 \) in contrast to all known structure functions of hadrons.

2. In higher order calculations (e.g. in the \( \overline{\text{MS}} \) scheme) the QCD scale is fixed so that \( \Lambda_{\overline{\text{MS}}} \) can in principle be determined from the magnitude of \( F_2^r \) at any \( x \) and \( Q^2 \). The prediction can be written as

\[
F_2^{\text{HO}}(x, Q^2) = \frac{3\alpha}{\pi} \sum q_i^4
\]

\[
\times \left[f(x) \ln(Q^2/\Lambda_{\overline{\text{MS}}}^2) + g(x) \ln(Q^2/\Lambda_{\overline{\text{MS}}}^2) + h(x)\right],
\]

(6)

where \( f(x), g(x), \) and \( h(x) \) are known functions of \( x \) [6,7]. \( F_2^{\text{HO}}(x, Q^2) \) is regular and positive for \( 0.2 \lesssim x \lesssim 0.9 \).

Such a determination of \( \Lambda_{\overline{\text{MS}}} \) is based on the measurement of the total \( e^+ e^- \) cross section and is therefore independent of the analysis of particular event topologies. Moreover this cross section has a more sensitive dependence on \( \Lambda_{\overline{\text{MS}}} \) than the total \( e^+ e^- \) annihilation cross section. However, besides the point-like contribution \( F_2^{\text{LO}} \) also contains a part from the hadronic (i.e. bound \( q\bar{q} \)) photon coupling which is not calculable in perturbative QCD. At the currently available \( Q^2 \) this
hadronic part is not negligible, but can be either sepa-
rated by a study of the measured $Q^2$ evolution, which
requires high statistics data, or inferred from measure-
ments of the pion structure function.

Measurements of the photon structure function
based on low statistics have been published by the
PLUTO [8], JADE [9] and CELLO [10] experiments
at PETRA. They demonstrate the existence of a
point-like part in the $\gamma\gamma$ cross section in addition to a
part due to the hadronic coupling of the photon. The
present paper describes a detailed measurement of $F^2$
as a function of $x$ and $Q^2$ based on 1417 hadronic
events which were collected from an integrated lumi-
nosity of 34.2 pb$^{-1}$ at the $e^+e^-$ storage ring PETRA
at DESY. In contrast to most of the previous investi-
gations, the data presented here have been unfolded to
correct for the resolution and acceptance of the experi-
ment.

For these measurements the PLUTO detector [3,
11] was extended by the addition of two magnetic
spectrometers covering the forward and backward
region ($4^\circ$ to $15^\circ$ and $165^\circ$ to $176^\circ$ with respect to
the $e^+e^-$ beam axis and $85\%$ of the azimuthal angle),
in which hadrons, photons, muons and electrons were
detected. Each spectrometer included both drift cham-
bers and shower counters, the latter called the “large
angle taggers” (LAT). The electron or positron scat-
tered at the larger angle was thereby reconstructed
with a resolution $\sigma(Q^2)/Q^2 = 10\%$. The data were
taken with a trigger which required only the deposi-
tion of shower energy $>4$ GeV in one of the LAT’s,
and no other condition. The resulting sample of
“single tag” multihadron events is therefore free of
trigger bias.

The following event selection criteria were defined
so that a good compromise between large acceptance
and low background contamination was achieved:

1. Tag definition. One isolated energy cluster in the
LAT with $E > 8$ GeV was required to be associated
with a reconstructed track in the forward spectrome-
ter drift chambers. To avoid edge effects the position
of the shower was restricted to a fiducial area corre-
sponding to an angular range $5^\circ < \Theta_1 < 15^\circ$.

2. Antitag condition. To keep the mass of the target
photons as small as possible, a veto against large $P^2$
(double tag) events was applied. No additional energy
cluster of more than 4 GeV was allowed in the small
angle tagger SAT, which covered the angular range
$1.5^\circ(178.5^\circ)$ to $4^\circ(176^\circ)$, or in the LAT.

3. Hadronic final state. A multihadronic final state
was required to have either 2 charged particles (tracks)
and $>2$ showers, or $>3$ tracks. The visible invariant
mass $W_{vis}$, reconstructed from the measured charged
and neutral particles in the final state, was required to
be between 1 and 12 GeV to maintain a good accept-
tance and to eliminate $e^+e^-$ annihilation events.

4. QED rejection. To reduce the background from
leptonic QED processes, all events with $<3$ tracks and
$>3$ showers were rejected in which one particle quali-
fied as an electron by having a track with momentum
$>1$ GeV and a shower cluster of energy
$>1$ GeV.

To determine the background contamination in the
selected event sample the following sources were con-
sidered:

(a) Beam-gas events.
(b) $1 \gamma$ annihilation events,
\[ e^+e^- \rightarrow \text{hadrons}, \]
\[ e^+e^- \rightarrow \tau\tau. \]
(c) $2 \gamma$ QED events,
\[ \gamma\gamma^* \rightarrow e^+e^-, \quad (\gamma^* = \text{tagged photon}), \]
\[ \gamma\gamma^* \rightarrow \tau\tau, \]
\[ \gamma\gamma \rightarrow \tau\tau, \text{ one } \tau \rightarrow e\nu. \]
(d) $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ via inelastic Compton
scattering [12].

The beam—gas background was determined from
the data using the side bands of the event vertex distri-
bution along the beam ($|z| > 40$ mm), and found to
be small ($4\%$). The $1\gamma$ annihilation background is esti-
mated to be less than $1\%$ mainly because of the re-
quirement of a high energy electron (“tag”) identified
by both a track and a shower in a forward spectrome-
ter. The QED processes (c) were studied in detail in a
Monte Carlo (MC) simulation. The most important
one is $\tau\tau$ production in the single tag mode ($5.9\%$).
The Compton process (d), which is in principle indis-
tinguishable from the genuine two photon process, was
calculated with a MC program to be $<1\%$. The sum of
all background sources (a)–(d) was calculated to be
$11\%$ of the selected event sample and was subtracted
from it.

The resulting data sample covers the $Q^2$ range $1.5
< Q^2 < 16$ GeV$^2$ with the average $Q^2 = 5.3$ GeV$^2$. The
antitag condition (with $\Theta_2^{\text{max}} \approx 30$ mrad) restricts
the target photon mass ($-P^2$)$^{1/2}$ to be small. The good
angular coverage of the detector for both charged and
neutral particles gives rise to a good event acceptance:
the fraction of the triggered events which are selected
for the final analysis rises from 45% at \( W = 2 \text{ GeV} \) to
70% at \( W = 10 \text{ GeV} \).

The evaluation of \( F_2(x, Q^2) \) is complicated for
several reasons. Firstly, due to particle losses and the
finite resolution of the detector the visible invariant mass \( W_{\text{vis}} \) is on average 24% lower than the true invari-
ant mass \( W \), with an r.m.s. resolution \( \sigma_W/W_{\text{vis}} = 27\% \).

Fig. 1a shows the corresponding mapping from true \( x \)
to \( x_{\text{vis}} \) determined from a MC simulation of the experi-
ment (see below), where \( x_{\text{vis}} \) is calculated with \( W_{\text{vis}} \) in
eq (3). It demonstrates that both the resolution in
\( x_{\text{vis}} \) is sufficiently small and the correlation between \( x \)
and \( x_{\text{vis}} \) is sufficiently defined for a meaningful inver-
sion of the mapping. Secondly, the accepted \( Q^2 \) range
varies with \( x \) because the measurements are restricted
to a fixed \( W \) interval. Thus in order to determine the
\( x \) dependence of \( F_2(x, Q^2) \) in such a way that it can
be compared easily with theoretical calculations, it is
necessary to interpolate the data in \( Q^2 \), and to present
\( F_2(x) \) at fixed \( Q^2 \) values which are independent of \( x \).

Thirdly, the multihadron acceptance after the data
selection depends on the fragmentation of the \( \gamma \gamma \) sys-
tem into hadrons.

These complications were overcome by determin-
ing \( F_2(x, Q^2) \) from the data by means of an unfolding
procedure. To this end a model for the fragmentation
to hadrons of the \( \gamma \gamma \) system was developed which,
when included in a MC simulation of the experiment

\[ \langle n^\pm \rangle = 2.0 \sqrt{W}, \quad \langle n^0 \rangle = 1.3 \sqrt{W} \quad (W \text{ in GeV}) \]

For a given mean multiplicity the actual multiplicities
were selected according to KNO [13] distributions
similar to those found in \( e^+e^- \) annihilation [3]. The
inverse relative dispersion \( \chi = (n)/D \) was fixed to be
2.7 for charged pions, and 2.4 for neutral pions. The
distribution of the transverse momenta \( p_T \) of the
pions relative to the \( \gamma \gamma \) axis was best described by a
mixture of isotropic phase space (IPS) and limited \( p_T \)
phase space (LPS) \(^1\), which changes with \( w = (W/7 \text{ GeV})^2 \).

In the unfolding procedure the mapping from \( x \) to
\( x_{\text{vis}} \) is determined by the MC simulation which in-
cludes both the \( \gamma \gamma \) fragmentation and the detector
response. This mapping is then inverted in such a way
[14] as to avoid the enhancement of random fluctua-
tions which usually occur in the matrix inversion in
the procedure. The number of bins and the bin sizes
in \( x \) are chosen to keep the correlations between the
unfolded data points small. The \( x \) and \( Q^2 \) dependence
of \( F_2 \) is represented by a factorising ansatz

\[ F_2(x, Q^2) = F_2(x)[1 + b \ln(Q^2/Q_0^2)] \]

with \( b = 0.17 \). This ansatz improves the interpolation

\(^1\) LPS events were generated like IPS events but with an
additional weight factor \( \exp(-5\rho^2) \) for each pion.
of the data in $Q^2$, and is not sensitive to the particular choice of $b$. Setting $b = 0$ changes the resulting $F_2(x)$ of the full $Q^2$ range by 10%, and by only 2% in each of the three $Q^2$ sub-ranges (see below). A non-factorising form in which $b$ depends on both $x$ and $Q^2$ is not necessary to describe our data.

The reliability of the unfolding procedure for the PLUTO detector has been verified by simulating events with a model photon structure function (QPM with standard Field–Feynman quark fragmentation) and then extracting $F_2(x)$ using the unfolding procedure described above. Fig. 1b demonstrates that the reconstructed $F_2(x)$ agrees satisfactorily with the input $F_2(x)$ and that the all-pion fragmentation model described above can describe the final state hadron distributions without introducing gross systematic errors into the result.

To adjust fragmentation parameters and to test the fragmentation model, various distributions of the data were compared with the MC simulation (which used the structure function obtained from the unfolding procedure). As examples in fig. 2 the inclusive $p_T^2$ distribution of charged particles and the neutral energy distribution demonstrate that the fragmentation model describes the data well; the $x_{vis}$ and $Q^2$ distributions show that the solution achieved with the unfolding procedure is acceptable.

The sensitivity of the unfolded $F_2(x, Q^2)$ to the details of the hadron fragmentation was studied by varying the fragmentation parameters. With the constraints of the distributions of final state hadrons, the variety of possible fragmentation models is restricted, and the uncertainty in the acceptance is estimated to be $<10\%$ for $x > 0.1$ and $<20\%$ below $\frac{1}{2}$. A more detailed discussion of this source of systematic errors is given in ref. [16].

A small fraction of the data is expected to originate from the c-quark component of the photon for which the fragmentation model above and the factorisation ansatz for $F_2(x, Q^2)$ are not appropriate. We therefore generated MC events for a c-quark mass of 1.6 GeV according to the QPM, with standard Field–Feynman fragmentation including strange particles, and subtracted them like a background ($\approx 10\%$) before applying the unfolding procedure. The resulting $F_2$ was then finally adjusted by adding the QPM charm contribution calculated at the $Q^2$ to which the

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![Graph](image-url)

Fig. 2. Distributions of experimental variables (points) compared with the Monte Carlo calculation (histogram) obtained with the extracted structure function $F_2(x, Q^2)$ and the fragmentation model described in the text. (a) Inclusive $p_T$ (transverse momentum relative to the beam axis), (b) inclusive neutral energy, (c) $x_{vis}$ and (d) $Q^2$.

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![Graph](image-url)

Fig. 3. The $x$ dependence of the structure function $F_2^2$ measured using data from three separate ranges of $Q^2$ and interpolated in the unfolding procedure to the fixed values of 2.4, 4.3 and 9.2 GeV$^2$.

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*If the same fragmentation model is used as in our analysis of jet production in $\gamma\gamma$ interactions [15] the result of the unfolding for $F_2^2$ agrees to well within the systematic errors quoted.*
Table 1

$x$ and $Q^2$ dependence of the photon structure function $F_2$.

<table>
<thead>
<tr>
<th>$Q^2$ (GeV$^2$)</th>
<th>$x$</th>
<th>$F_2/a$</th>
<th>$F_2/a -$charm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$0.016-0.110$</td>
<td>$0.204 \pm 0.014$</td>
</tr>
<tr>
<td>(1.5–3)</td>
<td>$0.110-0.370$</td>
<td>$0.272 \pm 0.026$</td>
<td>$0.263 \pm 0.026$</td>
</tr>
<tr>
<td></td>
<td>$0.370-0.700$</td>
<td>$0.222 \pm 0.064$</td>
<td>$0.222 \pm 0.064$</td>
</tr>
<tr>
<td>4.3</td>
<td>$0.030-0.170$</td>
<td>$0.256 \pm 0.014$</td>
<td>$0.218 \pm 0.014$</td>
</tr>
<tr>
<td>(3–6)</td>
<td>$0.170-0.440$</td>
<td>$0.295 \pm 0.020$</td>
<td>$0.273 \pm 0.020$</td>
</tr>
<tr>
<td></td>
<td>$0.440-0.800$</td>
<td>$0.336 \pm 0.044$</td>
<td>$0.336 \pm 0.044$</td>
</tr>
<tr>
<td>9.2</td>
<td>$0.060-0.230$</td>
<td>$0.354 \pm 0.027$</td>
<td>$0.300 \pm 0.027$</td>
</tr>
<tr>
<td>(6–16)</td>
<td>$0.230-0.540$</td>
<td>$0.402 \pm 0.029$</td>
<td>$0.340 \pm 0.029$</td>
</tr>
<tr>
<td></td>
<td>$0.540-0.900$</td>
<td>$0.492 \pm 0.069$</td>
<td>$0.492 \pm 0.069$</td>
</tr>
<tr>
<td>5.3</td>
<td>$0.035-0.072$</td>
<td>$0.245 \pm 0.015$</td>
<td>$0.216 \pm 0.015$</td>
</tr>
<tr>
<td>(1.5–16)</td>
<td>$0.072-0.174$</td>
<td>$0.307 \pm 0.010$</td>
<td>$0.258 \pm 0.010$</td>
</tr>
<tr>
<td></td>
<td>$0.174-0.319$</td>
<td>$0.377 \pm 0.025$</td>
<td>$0.222 \pm 0.025$</td>
</tr>
<tr>
<td></td>
<td>$0.319-0.490$</td>
<td>$0.329 \pm 0.037$</td>
<td>$0.329 \pm 0.037$</td>
</tr>
<tr>
<td></td>
<td>$0.490-0.650$</td>
<td>$0.439 \pm 0.052$</td>
<td>$0.439 \pm 0.052$</td>
</tr>
<tr>
<td></td>
<td>$0.650-0.840$</td>
<td>$0.361 \pm 0.076$</td>
<td>$0.361 \pm 0.076$</td>
</tr>
</tbody>
</table>

The charm contribution has been interpolated in the unfolding. As the charm contribution is small, the final result depends only slightly on this treatment of charm production and the changes are negligible if we use the all-pion fragmentation model above for the entire data.

Fig. 3 shows the structure function $F_2^o(x, \langle Q^2 \rangle)$ unfolded in three separate $Q^2$ intervals:

(a) $1.5 < Q^2 < 3$ GeV$^2$, \( \langle Q^2 \rangle = 2.4 \) GeV$^2$,
(b) $3 < Q^2 < 6$ GeV$^2$, \( \langle Q^2 \rangle = 4.3 \) GeV$^2$,
(c) $6 < Q^2 < 16$ GeV$^2$, \( \langle Q^2 \rangle = 9.2 \) GeV$^2$.

The rise of $F_2^o$ with increasing $x$ and $Q^2$, characteristic of the point-like $\gamma q$ coupling, is evident. In table 1 we also include $F_2^o$ values with the charm contribution subtracted by the method described above. Clearly the difference is small. The subtracted values can be compared directly to models using only $u$, $d$ and $s$ quarks.

The structure function determined from the full $Q^2$ range $1.5 < Q^2 < 16$ GeV$^2$ is shown in fig. 4. It is interpolated to $\langle Q^2 \rangle = 5.3$ GeV$^2$ in the unfolding. In addition to the statistical errors shown in figs. 3 and 4 and in table 1, there are systematic errors, arising mainly from the sensitivity of the acceptance calculation to hadron fragmentation as discussed above. The systematic errors due to the (non zero) target photon mass squared $-p^2$ and to the hadronic QCD corrections have also been studied and found to be small ($< 5\%$ in total). Including all contributions we estimate the systematic error of the data points to be $15\%$ for $x > 0.2$, and $25\%$ below. The systematic error of the average $F_2^o$ between $x = 0.3$ and $x = 0.8$ is $10\%$.

A determination of the QCD parameter $\Lambda_{\overline{MS}}$ from these results is not straightforward. A completely
model-independent determination based only on the $Q^2$ evolution of perturbative QCD requires significantly higher statistics and a larger $Q^2$ range. To make use of the sensitivity of the magnitude of $F_2^\gamma(x)$ on $\Lambda_{\overline{MS}}$ requires an assumed form for the non-perturbative QCD contributions.

One particular approach is to compare $F_2^\gamma(x)$ with the sum of a higher order perturbative QCD calculation and a part due to the hadronic photon coupling. The latter is estimated [12,17] from the pion structure function measured in the Drell-Yan process [18] to be

$$F_2^{\text{had}}/\alpha = (0.2 \pm 0.05)(1 - x).$$

In fig. 4 the sum of this hadronic part, of the QCD next to leading order calculation [6,19] using only u, d and s quarks, and of the charm contribution from the QPM is shown for three different values of $\Lambda_{\overline{MS}}$. For $x > 0.2$ both the x dependence and the absolute normalization of $F_2^\gamma$ are well described if a value for $\Lambda_{\overline{MS}}$ of about 200 MeV is used. Fig. 4 further demonstrates the sensitivity of the photon structure function $F_2^\gamma(x)$ to $\Lambda_{\overline{MS}}$. A fit in the interval $0.3 < x < 0.8$ yields $\Lambda_{\overline{MS}} = 190^{+50}_{-40}(\text{stat.})^{+60}_{-50}(\text{syst.})$ MeV.

Whether or not such a determination involves questionable theoretical assumptions is currently a source of much debate [20]. To minimise any sensitivity to details of the model for $F_2^{\gamma\text{had}}$ we have derived limits for $\Lambda_{\overline{MS}}$ from our results for $F_2^\gamma(x)$ at $Q^2 = 5.3$ GeV$^2$ making only weak theoretical assumptions. If we take as upper and lower limits $F_2^{\gamma\text{had}}/\alpha = 0.2$ (i.e. we assume there to be no decrease with increasing $x$ of $F_2^{\gamma\text{had}}$ from its value at small $x$) and $F_2^{\gamma\text{had}}/\alpha = 0$ (i.e. we consider $F_2^\gamma$ to be already asymptotic at $Q^2 = 5.3$ GeV$^2$) we find the respective limits $65 < \Lambda_{\overline{MS}} < 575$ MeV (90% confidence). They correspond to limits on $\alpha_s$ of $0.115 < \alpha_s < 0.170$ when extrapolated to $Q^2 = (35 \text{ GeV})^2$, characteristic of $e^+e^-$ annihilation at PETRA.

The $Q^2$ dependence of the data in table 1 and fig. 3 is consistent with the perturbative QCD expectation. A detailed QCD analysis of these and additional data extending up to $Q^2 = 100$ GeV$^2$ will be presented in a forthcoming paper.

To conclude, the photon structure function $F_2^\gamma(x, Q^2)$ has been measured for three $Q^2$ values in the range $1.5 < Q^2 < 16$ GeV$^2$. For $x > 0.2$ the results are well described by the sum of hadronic (VDM-like) and point-like coupling of the photon to hadrons, the latter calculated in next to leading order QCD. With weak theoretical assumptions we find bounds on the QCD scale parameter of $65 < \Lambda_{\overline{MS}} < 575$ MeV (90% confidence).

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