

Total cross sections

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Regge theory provides a very simple and economical description of all total cross sections.

The latest Review of Particle Properties [1] displays plots of a number of hadronic total cross sections and quotes fits to them of the form

$$\sigma^{\text{TOT}} = A + BP^n + C(\log P)^2 + D \log P. \quad (1)$$

V. Flaminio has kindly made available to us computer listings of the data on which these fits are based, and so we have taken the opportunity to make alternative fits to the data. Our fits are of Regge type, and therefore they have a sounder theoretical basis [2]. For the nine cross sections we consider we use only 16 free parameters, whereas fits of the type (1) need 45, but in the high-energy region where Regge theory is applicable (\sqrt{s} greater than about 5 or 10 GeV) our χ^2 are much the same. Further, as we discuss, it is interesting to try to understand why the parameters in the Regge fits take the values that they do, but it is difficult to attach much meaning to the parameters in the fits (1).

Our Regge fits are a sum of two powers:

$$\sigma^{\text{TOT}} = Xs^\epsilon + Ys^{-\eta}, \quad (2)$$

where we identify the first term as arising from pomeron exchange and the second from ρ , ω , f , a ex-

change. Because the pomeron has the quantum numbers of the vacuum, its couplings to a particle a and its antiparticle \bar{a} are equal, so that the values of the coefficient X should be equal for $\sigma(ab)$ and $\sigma(\bar{a}b)$. We emphasise that both ϵ and η are *effective* powers: they vary slowly with s . However the data indicate that this variation is very slow, and here we treat them as constant. From our previous work [3] we expect that the value of ϵ should be close to 0.08 and that η is about $\frac{1}{2}$.

The data are most extensive for pp and $\bar{p}p$. We first made a simultaneous fit to the pp and $\bar{p}p$ data for $\sqrt{s} > 10$ GeV, requiring the same values of X , ϵ and η for both reactions; that is, we have five parameters for the two reactions (just half the number required with a fit of the type (1)). This gave

$$\epsilon = 0.0808, \quad \eta = 0.4525,$$

and we use the *same* values of these two parameters for all our subsequent fits. For each other pair of reactions, ab and $\bar{a}b$, we have three new free parameters: a common value of X , and a Y for each. For the other reactions, we go down to $\sqrt{s} = 6$ GeV with our fitting procedure, because there are fewer available data points. (It would be very nice to have more measurements at higher energies for a variety of beams.) The values we need for X and Y in each case are shown in fig. 1. We have given each parameter to

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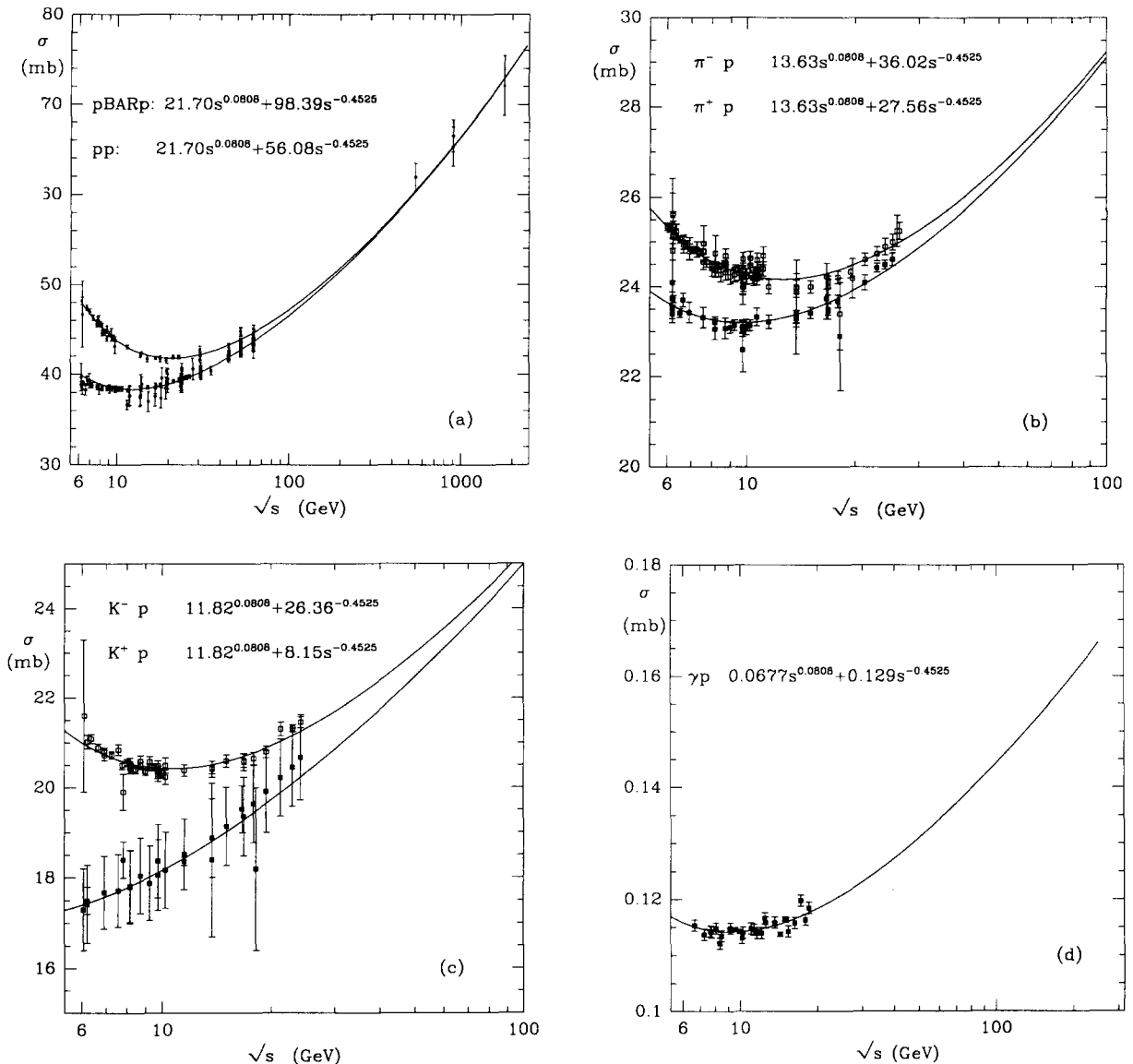


Fig. 1. Data for total cross sections with fits of type (2).

high accuracy because a small change in any one parameter has a significant effect on the χ^2 of the fit. This does not mean that the parameters are determined to that accuracy, because the χ^2 minimum is not very sharp and a change in any one parameter can be compensated by changes in all the others in such a way as not to have much effect on the χ^2 . Notice also that the quality of the data is such that the precise values of our parameters should not be taken

too seriously. This is particularly true of the coefficient Y for $\sigma(K^+p)$.

Our fits are compared with the data in figs. 1a–1e. We have a number of comments on these fits:

(1) Because the pp and $p\bar{p}$ data contain such a large number of points, the $p\bar{p}$ measurements at the CERN collider and at the Tevatron contribute rather little to the χ^2 per degree of freedom. So it is a definite success of the parametrisation that it agrees with the

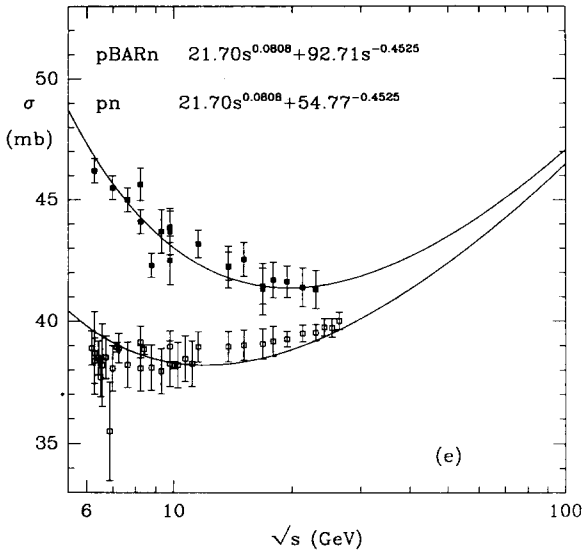


Fig. 1. Continued.

Tevatron measurement. Indeed we predicted in 1985 [4] that the cross section at $\sqrt{s}=1800$ GeV would be about 73 mb, which contrasts with many other predictions that were 10 or more mb higher [5].

(2) Notice that the rising component Xs^ϵ is present already at rather low energies, $\sqrt{s}=5$ GeV or so. Its form is unaffected by the onset at higher energies of new production processes, such as charm or minijets. Although at Tevatron energy there is undoubtedly a large amount of minijet production, other processes are reduced so as to compensate for this and there is no noticeable overall change in the rate of rise of the total cross section. This is not unexpected [6].

(3) The term Xs^ϵ corresponds to pomeron exchange. A simple Regge pole with a trajectory whose intercept is $\alpha(0)$ contributes a power $s^{\alpha(0)-1}$. Our s^ϵ is an effective power, with ϵ a little less than $\alpha(0) - 1$ because the term includes also the effect of the exchange of more than one pomeron. These multiple exchanges make the effective power ϵ reduce as the energy increases [2], but the data indicate that it does so only very slowly and so the effective power is almost constant. The combined effect of the multiple exchanges is surely rather small, otherwise we should not find that the same effective power ϵ fits all the different cross sections. We have previously [4] estimated that at ISR energies the multiple exchanges contribute only a few percent to the total cross sec-

tion, but they do have an important effect on the shape of the elastic differential cross section plotted against t . It was pointed out by Collins and Gault [7] that an almost-constant power causes no problem with the Froissart–Martin bound, because at present energies that bound is about 10 b, way above the data. Indeed, our fit $98.39s^{0.0808}$ to the rising part of $\sigma(\bar{p}p)$ exceeds $(\pi/m_\pi^2) \log^2(s/s_0)$, for any reasonable value of the unknown scale s_0 , only when $\sqrt{s} > 10^{24}$ GeV. Nevertheless we should emphasise that ϵ is not quite constant; if we pretend that it is, our fit predicts that the cross section at SSC energy ($\sqrt{s}=40$ TeV) is 120 mb, but because in reality ϵ is a little smaller by this energy this may be an over-estimate by a few mb.

(4) Because the Xs^ϵ represents pomeron exchange, which has vacuum quantum numbers, we have constrained our fits by requiring that the coefficient X be the same for $\sigma(ab)$ and $\sigma(\bar{a}b)$ ^{#1}. But it should also be the same for $\sigma(pn)$ as for $\sigma(pp)$. We have made a fit with $X_{pn}=X_{pp}$ not constrained by this latter requirement and obtained the value 22.15, almost equal to X_{pp} . Fig. 1e shows the fit with X_{pn} and X_{pp} constrained to be equal to X_{pp} .

(5) We find that the ratio of the coefficients X for πp and pp scattering is 0.63, close to the value $\frac{2}{3}$ of the additive-quark rule. This property has played an important role in the theory and the phenomenology of the pomeron [8]. Notice that the additive-quark rule should not be exact, but should receive corrections from hadron-radius effects [9] and, more importantly, from the fact that the $Xs^{0.0808}$ terms represent not only single-pomeron exchange, but also include some contribution from multiple exchanges.

(6) The value of X for $\sigma(Kp)$ is a little smaller than for $\sigma(\pi p)$; it is not understood whether this is because the pomeron coupling to strange quarks is only about 75% of its coupling to nonstrange quarks, or whether instead [9,10] it is important that the radius of the K is smaller than that of the pion. However, it is interesting that the additive-quark rule does seem to work when hadrons carrying strangeness are involved. Since $\Sigma^- = sdd$ and $\Xi^- = ssd$, in the combination

^{#1} This contrasts markedly with the fits of the type (1) reported in the Review of Particle Properties [1], which violate the Pomernanchuk theorem rather badly and would have the pp cross section overtaking the $\bar{p}p$ cross section at about $\sqrt{s}=100$ GeV.

$$2\sigma(\Xi^-p) - \sigma(\Sigma^-p) \tag{3}$$

only su and sd scattering is involved. Because of Zweig's rule, the only exchange that couples both to the s quark and to u or d is the pomeron. Hence the additive-quark rule predicts that (3) should have the high-energy behaviour $X_s s^{0.0808}$ with

$$X_s = X_{pp} - 3(X_{\pi p} - X_{Kp}) = 16.26. \tag{4}$$

For $\sqrt{s} = 14$ GeV this predicts a value 24.9 mb, while experiment gives [1] 25 ± 0.7 mb. For $\sqrt{s} = 16$ GeV the prediction is 25.5 mb, while the data give 24.6 ± 0.7 mb.

(7) Data for the ρ , ω , f, a trajectory are shown in fig. 2. The particles in square brackets are listed in the Review of Particle Properties [1], but their existence remains to be confirmed. A trajectory intercept of $\alpha(0)$ would yield a power η equal to $1 - \alpha(0)$. Previously [3] we fitted the trajectory by the straight line drawn in the figure, so giving a power η of 0.56, rather than the 0.4525 that we are now using. The implied adjustment to the line in fig. 2 is perfectly acceptable, though note that η , like ϵ , is only an effective power: it takes account not only of ρ , ω , f, a exchange, but also of the exchange of this trajectory combined with that of one or more pomerons. These extra exchanges tend to reduce the effective value of η .

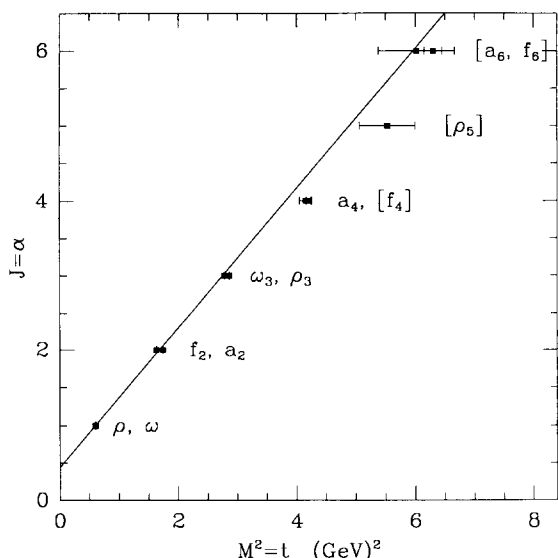


Fig. 2. The ρ , ω , f, a trajectory. The line is $\alpha(t) = 0.44 + 0.93t$.

(8) Our fit in fig. 1d to $\sigma(\gamma p)$ is similar to one we made of few years ago [11] when we considered at the same time data for low- Q^2 deep inelastic electron scattering. The prediction that at $\sqrt{s} = 200$ GeV the cross section is about 160 μb differs markedly from certain others in the literature [12] and will soon be tested at HERA.

(9) The fact that all total cross sections that have been measured rise with energy at the same rate s^ϵ makes it unnatural to attribute the rise to some intrinsic property of the hadrons involved. It is unhelpful to adopt a geometrical approach and to talk of hadrons becoming bigger and blacker as the energy increases. Rather, the rise is a property of something that is exchanged, the pomeron, and this is why the rise is universal. Hence, although our analysis superficially resembles that of Cheng, Walker and Wu long ago [13], we do not believe that it is correct to eikonalise in the way that they and others do. In this our conclusions are in accord with the recent important results from the UA8 Collaboration [14] at the CERN collider, which indicate that the pomeron does have a rather real existence: it can hit hadrons hard, break them up and knock most of their fragments sharply forward. In this respect, the pomeron resembles a hadron, and indeed UA8 have measured its structure function.

(10) Notice that the need for a substantial contribution to $\sigma(pp)$ from ρ , ω , f, a is in conflict with ideas about duality that were current 20 years ago. It is tempting to try to understand the relative magnitudes of the coefficients Y for the various cross sections by extending the additive-quark rule to the ρ , ω , f, a exchanges. However, if one determines their couplings by using the values we have found for Y_{pp} , $Y_{p\bar{p}}$, Y_{pn} and $Y_{\bar{p}n}$, then the predicted values of Y_{np} are too large by about 50%. A possible remedy is to assume that the proton wave function is not symmetric, but that rather a large fraction of its momentum is carried by one of its constituent u quarks. This has been suggested previously for other reasons [15]. We have not been able to reach any definite conclusion about this, because one would need information about the shapes of the momentum distributions and more accurate values of the coefficients Y than the data allow.

(11) Once one has fixed the magnitudes of the coefficients Y for a pair of total cross sections, one

knows the relative magnitudes of $C = +1$ f and a ω exchange, which contributes with equal signs to both, and $C = -1$ ρ and ω exchange, which contributes with opposite signs. Inserting Regge signature factors in the standard way [2], we may then deduce the ratio ρ of the real and imaginary parts of the forward elastic amplitude. We (and many others) have previously made the comparison with data for pp and $\bar{p}p$ scattering [4]. The only other pair of reactions for which there exist a reasonable quantity of data for ρ is $\pi^\pm p$ scattering [16]. The comparison of our analysis with these data is made in fig. 3. There are also a few data points for ρ in $K^\pm p$ scattering. As is seen in fig. 4, they have large errors. We have already remarked that Y_{K^+p} is not at all well determined by the data in fig. 1c and so our calculation shown in fig. 4 also has a large uncertainty. ρ_{K^-p} is particularly sensitive to Y_{K^+p} and can be increased significantly by reducing the latter but still keeping the fit to $\sigma(K^+p)$ within the experimental errors.

(12) There exist some data [16] for the quasi-elastic process

$\gamma p \rightarrow \phi p$.

Zweig's rule has the consequence that the only ex-

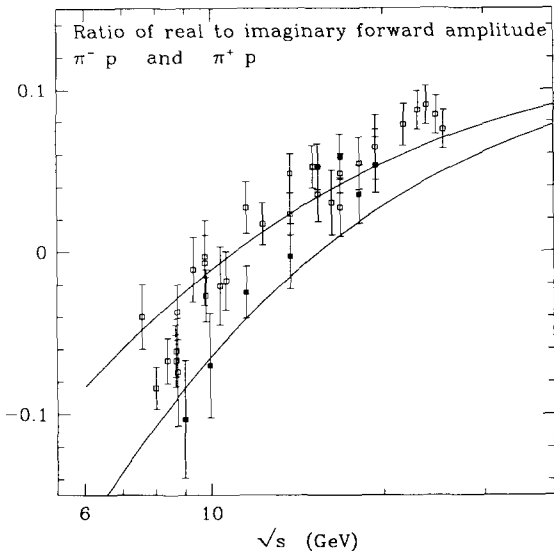


Fig. 3. Data for the ratio of the real and imaginary parts of the forward elastic amplitudes in πp scattering. The curves are deduced from the fits in fig. 1b.

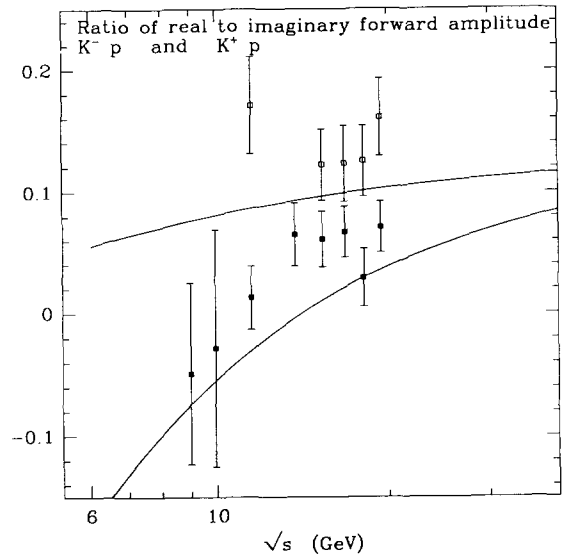


Fig. 4. Data for the ratio of the real and imaginary parts of the forward elastic amplitudes in Kp scattering. The curves are deduced from the fits in fig. 1c.

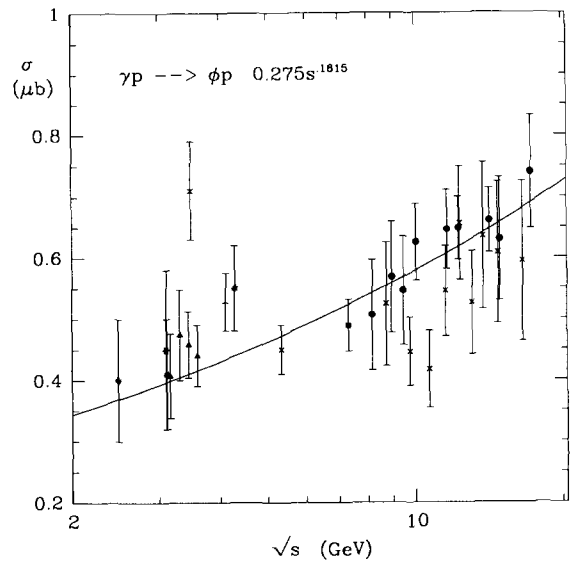


Fig. 5. Data for $\gamma p \rightarrow \phi p$ with curve corresponding to pomeron exchange.

change that couples both to the proton and to the γp transition vertex is the pomeron, so the cross section for this process should behave as $s^{2\epsilon}/b$, where b is the near-forward exponential t -slope. Fig. 5 shows a comparison with the data in the approximation that

b is constant (as both theory and experiment suggest). It would be good to have more data, not only for this reaction but also for $\gamma p \rightarrow J/\psi p$.

Conclusions. Regge theory remains one of the great truths of particle physics. We have shown how it provides an extremely simple and economical parametrisation of all total cross sections.

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References

- [1] Particle Data Group, K. Hikasa et al., Review of particle properties, Phys. Rev. D 45, No. 11 (1992).
- [2] P.D.B. Collins, Introduction to Regge theory (Cambridge U.P., Cambridge, 1977).
- [3] A. Donnachie and P.V. Landshoff, Nucl. Phys. B 231 (1983) 189.
- [4] A. Donnachie and P.V. Landshoff, Nucl. Phys. B 267 (1986) 690.
- [5] M. Block and R.N. Cahn, Phys. Lett. B 188 (1987) 143.
- [6] M. Jacob and P.V. Landshoff, Mod. Phys. Lett. A 1 (1987) 766.
- [7] P.D.B. Collins and F.D. Gault, Phys. Lett. B 273 (1978) 330.
- [8] P.V. Landshoff and J.C. Polkinghorne, Nucl. Phys. B 32 (1971) 541;
D.G. Richards, Nucl. Phys. B 258 (1985) 267.
- [9] P.V. Landshoff and O. Nachtmann, Z. Phys. C 35 (1987) 405;
J.F. Gunion and D. Soper, Phys. Rev. D 15 (1977) 2617.
- [10] B. Povh and J. Hüfner, Phys. Lett. B 245 (1990) 653;
H.G. Dosch, E. Ferreira and A. Krämer, preprint CERN-TH6454/92.
- [11] A. Donnachie and P.V. Landshoff, Nucl. Phys. B 244 (1984) 322.
- [12] R.S. Fletcher, T.K. Gaisser and F. Halzen, Phys. Rev. D 45 (1992) 377.
- [13] H. Cheng, J.K. Walker and T.T. Wu, Phys. Lett. B 44 (1973) 97.
- [14] P. Schlein (UA8 Collab.), in: Proc. LP THE'91 Conf. (Geneva, July 1991), ed. L. Di Lella (World Scientific, Singapore);
A. Brandt et al., to be submitted to Phys. Lett. B.
- [15] I.R. Zhitnitsky, A.A. Oglobin and V.L. Chernyak, Sov. J. Nucl. Phys. 48 (1988) 536.
- [16] Durham data base, telnet 129.234.8.100 – user PDG, password HEPDATA.