Modern Physics Letters A
Vol. 27, No. 13 (2012) 1230015 (22 pages)

DOI: 10.1142/S0217732312300157

NEUTRINOLESS DOUBLE-BETA DECAY. A BRIEF REVIEW

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Received 23 March 2012
Published 13 April 2012

In this brief review we discuss the generation of Majorana neutrino masses through the seesaw mechanism, the theory of neutrinoless double-beta decay, the implications of neutrino oscillation data for the effective Majorana mass, taking into account the recent Daya Bay measurement of ϑ_{13}, and the interpretation of the results of neutrinoless double-beta decay experiments.

Keywords: Neutrino mass; neutrino mixing; Majorana neutrinos; lepton number.

PACS Nos.: 14.60.Pq, 14.60.Lm, 14.60.St

1. Introduction

One of the most important recent discoveries in particle physics is the observation of neutrino oscillations in atmospheric,[1] solar,[2] reactor[3] and accelerator[4,5] neutrino experiments. Neutrino oscillations is a quantum-mechanical consequence of the neutrino mixing relation

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\nu_{iL}(x)\,, \qquad l = e, \mu, \tau\,. \tag{1}$$

Here $\nu_i(x)$ is the field of neutrinos with mass m_i, U is the 3×3 unitary PMNS[6–8] mixing matrix. The left-handed flavor field $\nu_{lL}(x)$ enters into the standard leptonic charged current

$$j_{\alpha}^{\mathrm{CC}}(x) = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\gamma_{\alpha} l_L(x) \tag{2}$$

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and determines the notion of a left-handed flavor neutrino ν_l which is produced in CC weak processes together with a lepton l^+. The flavor neutrino ν_l is described by the mixed state

$$|\nu_l\rangle = \sum_{i=1}^{3} U_{li}^* |\nu_i\rangle \,, \qquad (3)$$

where $|\nu_i\rangle$ is the state of a neutrino with mass m_i and a definite momentum.

The probability of the transition $\nu_l \to \nu_{l'}$ in vacuum is given by the standard expression (see Ref. 9)

$$P(\nu_l \to \nu_{l'}) = \left| \sum_i U_{l'i} e^{-iE_i t} U_{li}^* \right|^2 = \left| \delta_{l'l} + \sum_{i \neq k} U_{l'i} \left(e^{-i\frac{\Delta m_{ki}^2 L}{2E}} - 1 \right) U_{li}^* \right|^2 . \qquad (4)$$

Here $\Delta m_{ki}^2 = m_i^2 - m_k^2$, $L \simeq t$ is the distance between the neutrino detector and the neutrino source, and E is the neutrino energy.

In the standard parameterization, the 3×3 PMNS mixing matrix is characterized by three mixing angles, ϑ_{12}, ϑ_{23} and ϑ_{13}, by a Dirac CP-violating phase δ and by two possible Majorana CP-violating phases λ_2 and λ_3:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} D(\lambda_2, \lambda_3) \,, \qquad (5)$$

where $c_{ab} \equiv \cos\vartheta_{ab}$ and $s_{ab} \equiv \sin\vartheta_{ab}$. The diagonal matrix $D(\lambda_2, \lambda_3) = \mathrm{diag}(1, e^{i\lambda_2}, e^{i\lambda_3})$ is present only if massive neutrinos are Majorana particles. The Majorana phases have an effect in processes which are allowed only if massive neutrinos are Majorana particles and are characterized by a violation of the total lepton number, as neutrinoless double-beta decay (see Sec. 4). Since neutrino oscillations are flavor transitions without violation of the total lepton number, they do not depend on the Majorana phases.[10–13] The neutrino oscillation probabilities depend only on the four mixing parameters ϑ_{12}, ϑ_{23}, ϑ_{13} and δ, and on two independent mass-squared differences Δm_{12}^2 and Δm_{23}^2. From the analysis of the experimental data it follows that

$$\Delta m_{12}^2 \simeq \frac{1}{30} |\Delta m_{23}^2| \,. \qquad (6)$$

In the case of three-neutrino mixing assumed in Eqs. (1) and (3), two neutrino mass spectra are possible:

(1) Normal spectrum (NS)

$$m_1 < m_2 < m_3 \,, \qquad \Delta m_{12}^2 \ll \Delta m_{23}^2 \,. \qquad (7)$$

(2) Inverted spectrum (IS)

$$m_3 < m_1 < m_2 \,, \qquad \Delta m_{12}^2 \ll |\Delta m_{13}^2| \,. \tag{8}$$

The existing experimental data do not allow to establish, what type of neutrino mass spectrum is realized in nature.

Let us introduce the "solar" and "atmospheric" mass-squared differences Δm_s^2 and Δm_a^2, respectively. For both spectra we have $\Delta m_{12}^2 = \Delta m_s^2$. For normal (inverted) spectrum we have $\Delta m_{23}^2 = \Delta m_a^2$ ($|\Delta m_{13}^2| = \Delta m_a^2$).

From a three-neutrino analysis of the Super-Kamiokande data,[1] the values of the neutrino oscillation parameters in the case of a normal (inverted) mass spectrum are, at 90% C.L.,

$$\begin{gathered} 1.9(1.7)\cdot 10^{-3}\ \text{eV}^2 \le \Delta m_a^2 \le 2.6(2.7)\cdot 10^{-3}\ \text{eV}^2 \,, \\ 0.407 \le \sin^2\vartheta_{23} \le 0.583 \,, \qquad \sin^2\vartheta_{13} < 0.04(0.09) \,. \end{gathered} \tag{9}$$

The results of the Super-Kamiokande atmospheric neutrino experiment have been fully confirmed by the long-baseline accelerator neutrino experiments K2K[4] and MINOS.[5]

From the two-neutrino analysis of the MINOS $\nu_\mu \to \nu_\mu$ data, for the parameters Δm_a^2 and $\sin^2 2\vartheta_{23}$ the following values were obtained:

$$\Delta m_a^2 = (2.32^{+0.12}_{-0.08})\cdot 10^{-3}\ \text{eV}^2 \,, \qquad \sin^2 2\vartheta_{23} > 0.90 \,. \tag{10}$$

From the combined three-neutrino analysis of all solar neutrino data and the data of the reactor KamLAND experiment, it was found that[3]

$$\begin{aligned} \Delta m_s^2 &= (7.50^{+0.19}_{-0.20})\cdot 10^{-5}\ \text{eV}^2 \,, \\ \tan^2\vartheta_{12} &= 0.452^{+0.035}_{-0.033} \,, \\ \sin^2\vartheta_{13} &= 0.020 \pm 0.016 \,. \end{aligned} \tag{11}$$

From a similar analysis performed by the SNO collaboration, it was obtained that[2]

$$\begin{aligned} \Delta m_s^2 &= (7.41^{+0.21}_{-0.19})\cdot 10^{-5}\ \text{eV}^2 \,, \\ \tan^2\vartheta_{12} &= 0.446^{+0.030}_{-0.029} \,, \\ \sin^2\vartheta_{13} &= 0.025^{+0.018}_{-0.015} \,. \end{aligned} \tag{12}$$

The Daya Bay collaboration[14] measured recently with high precision the mixing angle ϑ_{13}:

$$\sin^2\vartheta_{13} = 0.024 \pm 0.004 \,. \tag{13}$$

This is a 5.2σ evidence of a nonzero value of ϑ_{13} which confirms the previous measurements of T2K,[15] MINOS[16] and Double Chooz.[17] It also confirms earlier indications of a nonzero value of ϑ_{13} found in the analysis of the data of solar and other neutrino experiments (see Eqs. (11) and (12) and Refs. 18–22). The Daya Bay measurement has important implications for theory[23] and experiment (see Ref. 24). It opens promising perspectives for the observation of CP violation in the lepton sector and matter effects in long-baseline experiments, which could allow to determine the character of the neutrino mass spectrum.

Several years ago an indication in favor of short-baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ transitions was found in the LSND experiment.[25] The LSND data can be explained by neutrino oscillations with $0.2\ \text{eV} < \Delta m^2 < 2\ \text{eV}$ and $10^{-3} < \sin^2 2\vartheta < 4 \cdot 10^{-2}$. Recently an additional (2σ) indication in favor of short-baseline oscillations, compatible with the LSND result, was obtained in the MiniBooNE experiment.[26] Moreover, the data obtained in old reactor short-baseline experiments can also be interpreted as indications in favor of oscillations[27] by using a new calculation of the reactor neutrino fluxes.[28,29] All these data (if confirmed) imply that the number of massive neutrinos is larger than three and in addition to the three flavor neutrinos ν_e, ν_μ, ν_τ mixed sterile neutrinos $\nu_{s_1}, \ldots$ must exist.

The problem of short-baseline neutrino oscillations and sterile neutrinos is a hot topic at the moment. Several new short-baseline reactor and accelerator experiments are aimed to check this possibility in the near future (see Ref. 30).

The absolute values of neutrino masses are currently unknown. The Mainz[31] and Troitsk[32] experiments on the high-precision measurement of the end-point part of the β-spectrum of ^{3}H decay found the 95% C.L. upper bounds

$$m_\beta \leq 2.3\ \text{eV (Mainz)}\,, \qquad m_\beta \leq 2.1\ \text{eV (Troitsk)}\,, \tag{14}$$

for the "average" neutrino mass (see Ref. 33)

$$m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2}\,. \tag{15}$$

From neutrino oscillation and tritium β-decay data we conclude that

(A) Neutrino masses are different from zero.
(B) Neutrino masses are much smaller than the masses of charged leptons and quarks.
(C) Neutrino masses are not (or not only) of Standard Model (SM) Higgs origin.

Several mechanisms of neutrino mass generation have been proposed. It is widely believed that the most plausible one is the seesaw mechanism.[34–37] According to this mechanism, small neutrino masses are generated by new interactions beyond the SM which violates the total lepton number L at a scale much larger than the electroweak scale $v = (\sqrt{2}G_{\text{F}})^{-1/2} \simeq 246$ GeV.

If the seesaw mechanism is realized, *the neutrinos ν_i with definite masses are Majorana particles* and, consequently, the lepton number violating neutrinoless double-beta decay ($0\nu\beta\beta$-decay) of even–even nuclei,

$$N(A,Z) \to N(A,Z+2) + e^- + e^-\,, \tag{16}$$

is allowed, where $N(A,Z)$ is a nucleus with nucleon number A and proton number Z. The knowledge of the nature of neutrinos with definite masses (Majorana or

Dirac?) is extremely important for the understanding of the origin of small neutrino masses. Using large detector masses, high energy resolutions and low backgrounds, the experiments on the search for neutrinoless double-beta decay allow to reach unparalleled sensitivities to extremely small effects due to the Majorana neutrino masses. In this brief review, we consider this process (see also Refs. 38–45).

2. Seesaw Mechanism of Neutrino Mass Generation

In this section, we briefly discuss the standard seesaw mechanism of neutrino mass generation.[34–37] We consider a general approach based on the effective Lagrangian formalism.[46] Let us assume that the Standard Model is valid up to some scale Λ. If we include effects of physics beyond the SM, the total Lagrangian (in the SM region) has the form

$$\mathcal{L}(\Lambda) = \mathcal{L}^{\mathrm{SM}} + \sum_{n\geq 1} \frac{1}{\Lambda^n} \mathcal{O}_{4+n} \,. \tag{17}$$

The second term is a nonrenormalizable part of the Lagrangian. It is built from SM fields and satisfies the requirement of $\mathrm{SU}(2)\times\mathrm{U}(1)$ invariance. The operator $\mathcal{O}_{4+n}$ has dimension M^{4+n}.

In the expansion (17) of the nonrenormalizable part of the Lagrangian in powers of $1/\Lambda$, the most important term for neutrino physics is the first one, $\mathcal{L}_I^{\mathrm{eff}} = \mathcal{O}_5/\Lambda$, which contains an operator of dimension five. This term can be built from the leptons and Higgs doublets:

$$\mathcal{L}_I^{\mathrm{eff}} = -\frac{1}{\Lambda}\sum_{l',l} [\bar{L}_{l'L}\tilde{H}] Y_{l'l} [\tilde{H}^T (L_{lL})^c] + \mathrm{h.c.}\,, \tag{18}$$

for $l, l' = e, \mu, \tau$. Here

$$L_{lL} = \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix}, \qquad H = \begin{pmatrix} H^{(+)} \\ H^{(0)} \end{pmatrix} \tag{19}$$

are the lepton and Higgs doublets, $\tilde{H} = i\tau_2 H^*$ is the conjugated Higgs doublet, $(L_{lL})^c = C(\overline{L}_{lL})^T$ is the (right-handed) charge-conjugated lepton doublet and $Y_{l'l} = Y_{ll'}$ are dimensionless constants (presumably of order one). Here C is the charge-conjugation matrix (which satisfies the relations $C\gamma_\alpha^T C^{-1} = -\gamma_\alpha$ and $C^T = -C$).

The Lagrangian (18) does not conserve the total lepton number L. Let us stress that this is the only Lagrangian term with a dimension-five operator which can be built with the SM fields.

The electroweak symmetry is spontaneously broken by the vacuum expectation value of the Higgs field

$$\tilde{H}_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}. \tag{20}$$

From Eqs. (18) and (20), we obtain the *left-handed Majorana neutrino mass term*

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\sum_{l',l}\bar{\nu}_{l'L}M^{L}_{l'l}(\nu_{lL})^{c} + \text{h.c.}\,, \tag{21}$$

where

$$M^{L}_{l'l} = \frac{v^2}{\Lambda}Y_{l'l}\,. \tag{22}$$

After the diagonalization of the symmetric matrix Y through the transformation

$$Y = UyU^{T}\,, \qquad U^{\dagger}U = 1\,, \qquad y_{ik} = y_i\delta_{ik}\,, \tag{23}$$

we obtain

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\sum_{i} m_i\bar{\nu}_i\nu_i\,. \tag{24}$$

Here

$$m_i = \frac{v^2}{\Lambda}y_i \tag{25}$$

and

$$\nu_i = \sum_{l} U^{\dagger}_{il}\nu_{lL} + \sum_{l}(U^{\dagger}_{il}\nu_{lL})^{c}\,. \tag{26}$$

From Eq. (26) it follows that the field ν_i satisfies the Majorana condition

$$\nu_i = \nu_i^{c} = C\bar{\nu}_i^{T}\,. \tag{27}$$

Thus, ν_i *is the field of the Majorana neutrino with mass* m_i given by Eq. (25).

From Eq. (26), one can see that the flavor field ν_{lL} is connected to ν_{iL} by the standard mixing relation

$$\nu_{lL} = \sum_{i} U_{li}\nu_{iL}\,, \tag{28}$$

where U is the unitary PMNS mixing matrix given in Eq. (5) in the standard parameterization, including the diagonal matrix of Majorana phases.

The values of the neutrino masses are determined by the seesaw factor v^2/Λ. Assuming that $m_3 \simeq 5\cdot 10^{-2}$ eV (which is the largest neutrino mass in the case of a neutrino mass hierarchy $m_1 \ll m_2 \ll m_3$), we have $\Lambda \simeq 10^{15}$ GeV. Thus, the standard seesaw mechanism of neutrino mass generation explains the smallness of neutrino masses by a violation of the total lepton number L in interactions due to physics beyond the SM at a very large (GUT) scale.

The local effective Lagrangian (18) can be obtained by considering the possible existence of heavy Majorana leptons N_i with masses $M_i \gg v$, which are singlets of the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge group of the SM. These heavy Majorana leptons can have the lepton number-violating Yukawa interaction with the standard lepton and Higgs doublets

$$\mathcal{L}_I^{Y} = -\sqrt{2}\sum_{i,l} Y_{li}\bar{L}_{lL}N_{iR}\tilde{H} + \text{h.c.} \tag{29}$$

At electroweak energies, the interaction (29) generates the effective Lagrangian (18) at second-order of perturbation theory. We have

$$\sum_i Y_{l'i} \frac{1}{M_i} Y_{li} = \frac{1}{\Lambda} Y_{l'l} \,. \tag{30}$$

From this relation it follows that the masses M_i determine the scale of new physics.

The seesaw mechanism based on the Lagrangian (29) is called "type I seesaw". There are two other well-studied[47] ways to generate the effective Lagrangian $\mathcal{L}_I^{\text{eff}}$ and, consequently, the left-handed Majorana mass term (21): through the interaction of the lepton and Higgs doublets with a heavy triplet scalar boson (type II seesaw) or with a heavy Majorana triplet fermion (type III seesaw).

Summarizing, if small neutrino masses are generated by the standard seesaw mechanism, we have the following consequences:

(1) Neutrinos with definite masses are truly neutral Majorana particles.
(2) Neutrino masses are given by the seesaw relation (25). Hence, the neutrino masses are suppressed with respect to the masses of charged leptons and quarks, which are proportional to v, by the small ratio v/Λ.
(3) The Majorana neutrino mass term is the only implication at the electroweak scale of a possible existence of heavy Majorana particles.
(4) CP-violating decays of heavy Majorana particles in the early Universe could be the origin of the baryon asymmetry of the Universe (see Ref. 48).

3. On the Theory of $0\nu\beta\beta$-Decay

In this section, we present a brief derivation of the matrix element of the neutrinoless double-beta decay process in Eq. (16), assuming that this process is induced by Majorana neutrino masses and mixing (see Refs. 49, 50 and 42).

The standard effective Hamiltonian of the process has the form

$$\mathcal{H}_I(x) = \frac{G_{\text{F}}}{\sqrt{2}} 2\bar{e}_L(x)\gamma_\alpha \nu_{eL}(x) j^\alpha(x) + \text{h.c.} \tag{31}$$

Here G_{F} is the Fermi constant and $j^\alpha(x)$ is the hadronic charged current which does not change strangeness. In terms of the quark fields, the current $j^\alpha(x)$ has the form

$$j^\alpha(x) = 2\cos\vartheta_C \bar{u}_L(x)\gamma^\alpha d_L(x) \,. \tag{32}$$

The mixed flavor field $\nu_{eL}(x)$ is given by the relation (28) with $l = e$:

$$\nu_{eL}(x) = \sum_i U_{ei} \nu_{iL}(x) \,, \tag{33}$$

where U is the PMNS mixing matrix and $\nu_i(x)$ is the field of the Majorana neutrino with mass m_i, which satisfies the Majorana condition (27).

The process (16) is of second-order in $G_{\rm F}$, with the exchange of virtual neutrinos. The matrix element of the process is given by

$$\langle f|S^2|i\rangle = -4\left(\frac{G_{\rm F}}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}\int d^4x_1\,d^4x_2\sum_i \bar{u}_L(p_1)e^{ip_1x_1}\gamma_\alpha U_{ei} \times \langle 0|T(\nu_{iL}(x_1)\nu_{iL}^T(x_2)|0\rangle\gamma_\beta^T U_{ei}\bar{u}_L^T(p_2)e^{ip_2x_2}\langle N_f|T(J^\alpha(x_1)J^\beta(x_2))|N_i\rangle\,. \tag{34}$$

Here p_1 and p_2 are electron four-momenta, $J^\alpha(x)$ is the hadronic charged current in the Heisenberg representation,[a] N_i and N_f are the states of the initial and final nuclei with respective four-momenta $P_i = (E_i,\,\mathbf{p}_i)$ and $P_f = (E_f,\,\mathbf{p}_f)$, and $N_p^{-1} = (2\pi)^{3/2}\sqrt{2p^0}$ is the standard normalization factor.

Taking into account the Majorana condition (27), for the neutrino propagator we find the expression[b]

$$\langle 0|T(\nu_{iL}(x_1)\bar{\nu}_{iL}(x_2))|0\rangle = -\frac{i}{(2\pi)^4}\int d^4q\,e^{-iq(x_1-x_2)}\frac{m_i}{q^2-m_i^2}\frac{1-\gamma_5}{2}C\,. \tag{35}$$

Performing the integration over x_1^0, x_2^0 and q^0 in Eqs. (34) and (35), the matrix element of the process takes the form

$$\begin{aligned}\langle f|S^2|i\rangle = {} & 2i\left(\frac{G_{\rm F}}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}2\pi\delta(E_f+p_1^0+p_2^0-E_i)\bar{u}(p_1)\gamma_\alpha\gamma_\beta(1+\gamma_5)C\bar{u}^T(p_2)\\ & \times\int d^3x_1\,d^3x_2\,e^{-i\mathbf{p}_1\mathbf{x}_1-i\mathbf{p}_2\mathbf{x}_2}\sum_j U_{ej}^2 m_j\int\frac{d^3q}{(2\pi)^3}\frac{e^{i\mathbf{q}(\mathbf{x}_1-\mathbf{x}_2)}}{q_j^0}\\ & \times\left(\sum_n\frac{\langle N_f|J^\alpha(\mathbf{x}_1)|N_n\rangle\langle N_n|J^\beta(\mathbf{x}_2))|N_i\rangle}{E_n+p_2^0+q_j^0-E_i-i\epsilon}\right.\\ & \left.+\sum_n\frac{\langle N_f|J^\beta(\mathbf{x}_2)|N_n\rangle\langle N_n|J^\alpha(\mathbf{x}_1))|N_i\rangle}{E_n+p_1^0+q_j^0-E_i-i\epsilon}\right)\end{aligned} \tag{36}$$

where $q_j^0 = \sqrt{|\mathbf{q}|^2+m_j^2}$ and E_n are the energy levels of the intermediate nuclear state.

This is an exact expression for the matrix element of $0\nu\beta\beta$-decay at second-order of perturbation theory. In the following we consider major $0^+\to 0^+$ transitions of even–even nuclei, for which the following standard approximations[49] apply:

[a]In Eq. (34) strong interactions are taken into account.
[b]The neutrino propagator is proportional to m_i. This is connected to the fact that only left-handed neutrino fields enter into the Hamiltonian of weak interactions. Thus, in the case of massless neutrinos the matrix element of neutrinoless double β-decay is equal to zero. This is a consequence of the general theorem on the equivalence of the theories with massless Majorana and Dirac neutrinos.[51,52]

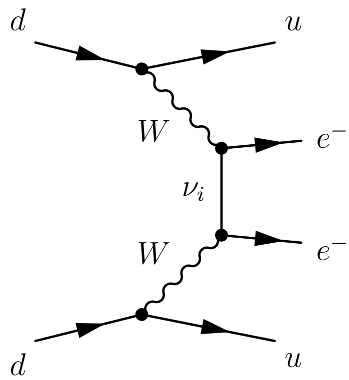

Fig. 1. Feynman diagram of the elementary particle transition which induces $0\nu\beta\beta$-decay.

(1) Effective Majorana mass approximation.
$0\nu\beta\beta$-decay is due to the exchange of virtual neutrinos (see the diagram in Fig. 1). Taking into account that the average distance between nucleons in a nucleus is about 10^{-13} cm, the uncertainty relation implies that the average neutrino momentum is $q \simeq 100$ MeV. On the other hand, from tritium experiments we have the upper bounds in Eq. (14), which constrain all the masses m_j to be smaller than about 2 eV. Therefore, the neutrino masses can be safely neglected in the denominators in Eq. (36) and we have $q_j^0 = \sqrt{|\mathbf{q}|^2 + m_j^2} \simeq q$, with $q = |\mathbf{q}|$.

Thus, from Eq. (36) it follows that in the matrix element of $0\nu\beta\beta$-decay *the neutrino properties and the nuclear properties are factorized and the neutrino masses and mixing enter into the matrix element in the form of the effective Majorana mass*

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i \,. \tag{37}$$

(2) Long-wavelength approximation.
We have $|\mathbf{p}_k \mathbf{x}_k| \leq |\mathbf{p}_k| R$ $(k = 1, 2)$, where $R \simeq 1.2\ A^{1/3} \cdot 10^{-13}$ cm is the radius of a nucleus with nucleon number A. Taking into account that $|\mathbf{p}_k| \lesssim 1$ MeV, we have $|\mathbf{p}_k \mathbf{x}_k| \ll 1$. Thus, we have $e^{-i\mathbf{p}_1\mathbf{x}_1 - i\mathbf{p}_2\mathbf{x}_2} \simeq 1$ (this approximation means that electrons are produced in S-states).

(3) Closure approximation.
The energy of the virtual neutrino, $q \simeq 100$ MeV, is much larger than the excitation energy $E_n - E_i$. Thus, the energy of the intermediate states E_n can be approximated by an average energy $\bar{E}$. In this approximation, called "closure approximation", we have

$$\frac{\langle N_f | J^\alpha(\mathbf{x}_1) | N_n \rangle \langle N_n | J^\beta(\mathbf{x}_2)) | N_i \rangle}{E_n + p_k^0 + q_j^0 - E_i - i\epsilon} \simeq \frac{\langle N_f | J^\alpha(\mathbf{x}_1) J^\beta(\mathbf{x}_2)) | N_i \rangle}{\bar{E} + p_k^0 + q - E_i - i\epsilon} \,. \tag{38}$$

Taking into account these approximations and considering commuting hadronic currents (see Eqs. (41) and (42) below), for the matrix element of $0\nu\beta\beta$-decay we obtain the expression

$$\begin{aligned}\langle f|S^{(2)}|i\rangle = 8\pi i\left(\frac{G_{\mathrm{F}}}{\sqrt{2}}\right)^2 m_{\beta\beta} N_{p_1} N_{p_2} \bar{u}(p_1)(1+\gamma_5)C\bar{u}^T(p_2) \\ \times \int d^3x_1\, d^3x_2 \langle N_f|J^\alpha(\mathbf{x}_1)K(|\mathbf{x}_1-\mathbf{x}_2|)J_\alpha(\mathbf{x}_2))|N_i\rangle \\ \times \delta(E_f + p_1^0 + p_2^0 - E_i)\,. \end{aligned} \tag{39}$$

Here

$$K(|\mathbf{x}_1-\mathbf{x}_2|) = \frac{1}{(2\pi)^3}\int d^3q \frac{e^{i\mathbf{q}(\mathbf{x}_1-\mathbf{x}_2)}}{q[\bar{E}+q-(M_i+M_f)/2]}\,, \tag{40}$$

where $M_i(M_f)$ is the mass of the initial (final) nucleus.

In the calculation of the hadronic part of the matrix element of $0\nu\beta\beta$-decay, the following approximate expression for the effective charged current $J^\alpha(\mathbf{x}) = (J^0(\mathbf{x}), \mathbf{J}(\mathbf{x}))$ is used[53]:

$$J^0(\mathbf{x}) = \sum_{n=1}^{A} \tau_n^+ \delta(\mathbf{x}-\mathbf{r}_n) g_V(q^2) \tag{41}$$

and

$$\mathbf{J}(\mathbf{x}) = -\sum_{n=1}^{A} \tau_n^+ \delta(\mathbf{x}-\mathbf{r}_n)\left[g_A(q^2)\boldsymbol{\sigma}_n + g_M(q^2) i\frac{\boldsymbol{\sigma}_n\times\mathbf{q}}{2m_p} - g_P(q^2)\frac{(\boldsymbol{\sigma}_n\cdot\mathbf{q})\mathbf{q}}{2m_p}\right]. \tag{42}$$

Here, σ_n^i and τ_n^i are Pauli matrices acting, respectively, on the spin and isospin doublets of the n nucleon, $\tau_+ = (\tau_1 + i\tau_2)/2$, $\mathbf{r}_n$ is the coordinate of the n nucleon, m_p is the proton mass, $g_V(q^2)$, $g_A(q^2)$, $g_M(q^2)$ and $g_P(q^2)$ are the vector, axial, magnetic and pseudoscalar weak form factors of the nucleon. From the conserved vector current (CVC) and partially conserved axial current (PCAC) hypotheses, it follows that

$$\begin{aligned} g_V(q^2) &= F_1^p(q^2) - F_1^n(q^2)\,, \\ g_M(q^2) &= F_2^p(q^2) - F_2^n(q^2)\,, \\ g_P(q^2) &= \frac{2m_p g_A}{q^2+m_\pi^2}\,, \end{aligned} \tag{43}$$

where $F_1^{p(n)}$ and $F_2^{p(n)}$ are the Dirac and Pauli electromagnetic form factors of the proton (neutron) and $g_A \simeq 1.27$ is the axial coupling constant of the nucleon.

The expressions (41) and (42) can be obtained from the one-nucleon matrix element of the hadronic charged current. For the number density of nucleons in a nucleus, the following approximate expression is used:

$$\bar{\Psi}(\mathbf{x})\gamma^0\Psi(\mathbf{x}) = \sum_{n=1}^{A}\delta(\mathbf{x}-\mathbf{r}_n)\,. \tag{44}$$

Table 1. The values of $G^{0\nu}(Q,Z)$, Q and natural abundance of the initial isotope for several $\beta\beta$-decay processes of experimental interest. Table adapted from Ref. 45.

$\beta\beta$-decay	$G^{0\nu}$ $[10^{-14}\ \mathrm{y}^{-1}]$	Q [keV]	Nat. abund. [%]	Experiments
$^{48}\mathrm{Ca} \to {}^{48}\mathrm{Ti}$	6.3	4273.7	0.187	CANDLES
$^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se}$	0.63	2039.1	7.8	GERDA, Majorana
$^{82}\mathrm{Se} \to {}^{82}\mathrm{Kr}$	2.7	2995.5	9.2	SuperNEMO, Lucifer
$^{100}\mathrm{Mo} \to {}^{100}\mathrm{Ru}$	4.4	3035.0	9.6	MOON, AMoRe
$^{116}\mathrm{Cd} \to {}^{116}\mathrm{Sn}$	4.6	2809	7.6	Cobra
$^{130}\mathrm{Te} \to {}^{130}\mathrm{Xe}$	4.1	2530.3	34.5	CUORE
$^{136}\mathrm{Xe} \to {}^{136}\mathrm{Ba}$	4.3	2461.9	8.9	EXO, KamLAND-Zen, NEXT, XMASS
$^{150}\mathrm{Nd} \to {}^{150}\mathrm{Sm}$	19.2	3367.3	5.6	SNO+, DCBA/MTD

The nuclear matrix element (NME) $M^{0\nu}$, which is the integrated product of two hadronic charged currents and a neutrino propagator, is a sum of a Fermi (F), a Gamow–Teller (GT) and a tensor (T) term:

$$M^{0\nu} = \langle 0_f^+ | \sum_{k,l} \tau_k^+ \tau_l^+ \left[\frac{H_{\mathrm{F}}(r_{kl})}{g_A^2} + H_{\mathrm{GT}}(r_{kl}) \boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_l - H_{\mathrm{T}}(r_{kl}) S_{kl} \right] |0_i^+\rangle \,. \quad (45)$$

Here $S_{kl} = 3(\boldsymbol{\sigma}_k \cdot \mathbf{r}_{kl})(\boldsymbol{\sigma}_l \cdot \mathbf{r}_{kl}) - \boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_l$, with $\mathbf{r}_{kl} = \mathbf{r}_k - \mathbf{r}_l$, and the neutrino potentials $H_{\mathrm{F,GT,T}}(r_{kl})$ are given by the expressions

$$H_{\mathrm{F,GT,T}}(r_{kl}) = \frac{2}{\pi} R \int_0^\infty \frac{j_{0,0,2}(q r_{kl}) h_{\mathrm{F,GT,T}}(q^2) q}{q + \bar{E} - (M_i + M_f)/2} dq \,, \quad (46)$$

where R is the radius of the nucleus, and the functions $h_{\mathrm{F,GT,T}}(q^2)$ are combinations of different form factors.[c]

Taking into account the Coulomb interaction of the electrons and the final nucleus, for the total width of $0\nu\beta\beta$-decay we find the general expression

$$\Gamma^{0\nu} = \frac{1}{T_{1/2}^{0\nu}} = G^{0\nu}(Q,Z) |M^{0\nu}|^2 \frac{|m_{\beta\beta}|^2}{m_e^2} \,, \quad (47)$$

where $G^{0\nu}(Q,Z)$ is a known integral over the phase space, $Q = M_i - M_f - 2m_e$ is the Q-value of the process, and m_e is the electron mass. The numerical values of $G^{0\nu}(Q,Z)$, Q and the natural abundance of several nuclei of experimental interest are presented in Table 1.

4. Effective Majorana Mass

The effective Majorana mass $m_{\beta\beta}$ is determined by the neutrino masses, the mixing angles and the Majorana phases. In this Section we discuss which are the possible

[c] The functions $h_{\mathrm{F,GT,T}}(q^2)$ can be found in Ref. 54.

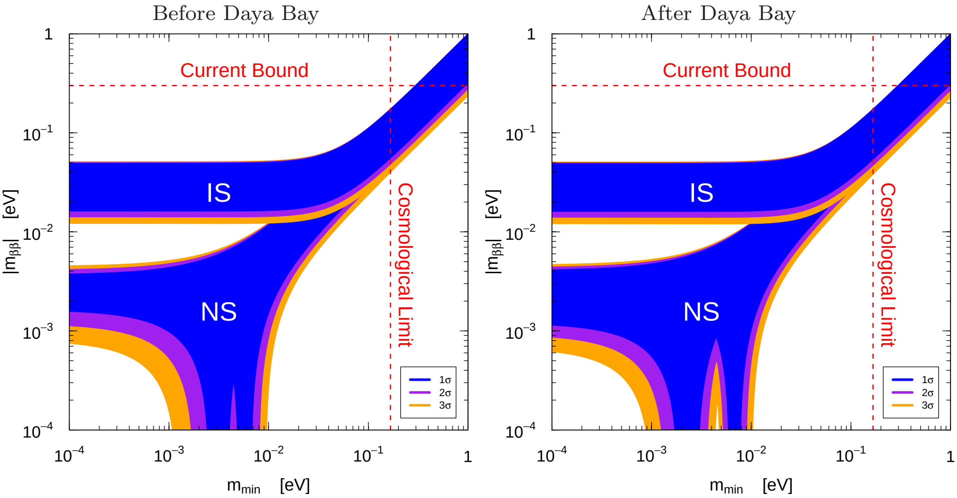

Fig. 2. Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of the lightest neutrino mass in the normal (NS, with $m_{\rm min} = m_1$) and inverted (IS, with $m_{\rm min} = m_3$) neutrino mass spectra before and after the Daya Bay[14] measurement of ϑ_{13} in Eq. (13). The current upper bound on $|m_{\beta\beta}|$ (see Eqs. (70), (72) and (74)) and the cosmological bound (see Ref. 55) on $\sum_i m_i \simeq 3m_{\rm min}$ in the quasi-degenerate region are indicated.

values of the effective Majorana mass which can be obtained taking into account the information on the neutrino mass-squared differences and mixing angles obtained from neutrino oscillation data.

In the standard parametrization (5) of the mixing matrix, we have

$$|m_{\beta\beta}| = |\cos^2\vartheta_{12}\cos^2\vartheta_{13}m_1 + e^{2i\alpha_{12}}\sin^2\vartheta_{12}\cos^2\vartheta_{13}m_2 + e^{2i\alpha_{12}}\sin^2\vartheta_{13}m_3|\,, \quad (48)$$

where α_{12} and α_{13} are, respectively, the phase differences of U_{e2} and U_{e3} with respect to U_{e1}: $\alpha_{12} = \lambda_2$ and $\alpha_{13} = \lambda_3 - \delta$ in the standard parametrization (5) of the mixing matrix. Therefore, $0\nu\beta\beta$-decay depends not only on the mixing angles and Dirac CP-violating phase, but also on the Majorana CP-violating phases. This is in agreement with the discussion after Eq. (5), since the total lepton number is violated in $0\nu\beta\beta$-decay.

In the case of a NS, the neutrino masses m_2 and m_3 are connected with the lightest mass m_1 by the relations

$$m_2 = \sqrt{m_1^2 + \Delta m_s^2}\,, \qquad m_3 = \sqrt{m_1^2 + \Delta m_s^2 + \Delta m_a^2}\,. \quad (49)$$

On the other hand, in a IS m_3 is the lightest mass and we have

$$m_1 = \sqrt{m_3^2 + \Delta m_a^2}\,, \qquad m_2 = \sqrt{m_3^2 + \Delta m_a^2 + \Delta m_s^2}\,. \quad (50)$$

Figure 2 shows the value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of the lightest neutrino mass in the normal and inverted neutrino mass spectra before and after the Daya Bay[14] measurement of ϑ_{13} in Eq. (13). We used the values of the neutrino oscillation parameters obtained in the global analysis presented in Ref. 56:

$$\begin{aligned} \Delta m^2_{12} &= 7.59^{+(0.20,0.40,0.60)}_{-(0.18,0.35,0.50)} \times 10^{-5}\ \text{eV}^2\,, \\ \sin^2\vartheta_{12} &= 0.312^{+(0.017,0.038,0.058)}_{-(0.015,0.032,0.042)}\,, \end{aligned} \tag{51}$$

and in the NS

$$\begin{aligned} \Delta m^2_{13} &= 2.50^{+(0.09,0.18,0.26)}_{-(0.16,0.25,0.36)} \times 10^{-3}\ \text{eV}^2\,, \\ \sin^2\vartheta_{13} &= 0.013^{+(0.007,0.015,0.022)}_{-(0.005,0.009,0.012)}\,, \end{aligned} \tag{52}$$

whereas in the IS

$$\begin{aligned} -\Delta m^2_{13} &= 2.40^{+(0.08,0.18,0.27)}_{-(0.09,0.17,0.27)} \times 10^{-3}\ \text{eV}^2\,, \\ \sin^2\vartheta_{13} &= 0.016^{+(0.008,0.015,0.023)}_{-(0.006,0.011,0.015)}\,. \end{aligned} \tag{53}$$

The three levels of uncertainties correspond to $(1\sigma, 2\sigma, 3\sigma)$. In the "After Daya Bay" plot in Fig. 2 we replaced the value of ϑ_{13} in Eqs. (52) and (53) with that measured by the Daya Bay Collaboration in Eq. (13). The uncertainties for $|m_{\beta\beta}|$ have been calculated using the standard method of propagation of uncorrelated errors, taking into account the asymmetric uncertainties in Eqs. (51)–(53).

In the following, we discuss the predictions for the effective Majorana mass in three cases with characteristic neutrino mass spectra:

(1) Hierarchy of neutrino masses[d]:

$$m_1 \ll m_2 \ll m_3\,. \tag{54}$$

(2) Inverted hierarchy of neutrino masses:

$$m_3 \ll m_1 \lesssim m_2\,. \tag{55}$$

(3) Quasi-degenerate neutrino mass spectrum:

$$\sqrt{\Delta m^2_{\text{a}}} \ll m_0 \simeq \begin{cases} m_1 \lesssim m_2 \lesssim m_3\,, & \text{(NS)}\,, \\ m_3 \lesssim m_1 \lesssim m_2\,, & \text{(IS)}\,, \end{cases} \tag{56}$$

where m_0 is the absolute mass scale common to the three masses. As one can see from Fig. 2, the Daya Bay measurement of ϑ_{13} has a visible impact on the value of $|m_{\beta\beta}|$ only in the case of a hierarchy of neutrino masses, discussed in the following, because only in that case the contribution of the largest mass m_3, which is weighted by $\sin^2\vartheta_{13}$, is decisive.

[d]Quarks and charged leptons have this type of mass spectrum.

4.1. *Hierarchy of neutrino masses*

In this case we have

$$m_1 \ll \sqrt{\Delta m_s^2}\,, \qquad m_2 \simeq \sqrt{\Delta m_s^2}\,, \qquad m_3 \simeq \sqrt{\Delta m_a^2}\,. \tag{57}$$

Thus, m_2 and m_3 are determined by the solar and atmospheric neutrino mass-squared differences. Neglecting the contribution of m_1 to the effective Majorana mass, from Eq. (48) we find

$$|m_{\beta\beta}| \simeq |\sin^2\vartheta_{12}\sqrt{\Delta m_s^2} + e^{2i\alpha_{23}}\sin^2\vartheta_{13}\sqrt{\Delta m_a^2}|\,, \tag{58}$$

where α_{23} is the phase difference between U_{e3} and U_{e2}: $\alpha_{23} = \alpha_{13} - \alpha_{12} = \lambda_3 - \delta - \lambda_2$ in the standard parameterization (5) of the mixing matrix.

The first term in Eq. (58) is small because of the smallness of Δm_s^2. On the other hand, the contribution of the "large" Δm_a^2 is suppressed by the small factor $\sin^2\vartheta_{13}$. Hence, both terms must be taken into account and cancellations are possible, as shown in Fig. 2.

As one can see from Fig. 2, in the case of a hierarchy of neutrino masses we have the upper bound

$$|m_{\beta\beta}| \le \sin^2\vartheta_{12}\sqrt{\Delta m_s^2} + \sin^2\vartheta_{13}\sqrt{\Delta m_a^2} \lesssim 5\cdot 10^{-3}\ \text{eV}\,, \tag{59}$$

which is significantly smaller than the expected sensitivity of the future experiments on the search for $0\nu\beta\beta$-decay (see Sec. 6). This bound corresponds to the case of $e^{2i\alpha_{23}} = 1$. It is slightly increased by the Daya Bay measurement of ϑ_{13} in Eq. (13), because the additive contribution of $\sin^2\vartheta_{13}\sqrt{\Delta m_a^2}$ in Eq. (58) is increased. On the other hand, one can see from Fig. 2 that the lower bound on $|m_{\beta\beta}|$ for $m_1 \ll 10^{-3}$ eV, which corresponds to $e^{2i\alpha_{23}} = -1$, is slightly decreased by the Daya Bay measurement of ϑ_{13}, because the increased contribution of $\sin^2\vartheta_{13}\sqrt{\Delta m_a^2}$ in this case is subtracted.

From Fig. 2 one can also see that when the contribution of m_1 is not negligible, there can be cancellations among the three mass contributions. The two extreme cases in which cancellations can happen are the following ones in which CP is conserved:

$e^{2i\alpha_{12}} = -1$ and $e^{2i\alpha_{13}} = +1$. The value of m_1 for which cancellations suppress $|m_{\beta\beta}|$ is slightly decreased by the Daya Bay measurement of ϑ_{13}, because the larger value of $\sin^2\vartheta_{13}m_3$ adds to the contribution of m_1. Hence, a smaller value of m_1 is required to cancel the sum of the contributions of m_1 and m_3 with the opposite contribution of m_2.

$e^{2i\alpha_{12}} = -1$ and $e^{2i\alpha_{13}} = -1$. The value of m_1 for which cancellations suppress $|m_{\beta\beta}|$ is slightly increased by the Daya Bay measurement of ϑ_{13}, because the larger value of $\sin^2\vartheta_{13}m_3$ adds to the contribution of m_2. Hence, a larger value of m_1 is required to cancel the contribution of m_1 with the opposite sum of contributions of m_2 and m_3.

Figure 2 shows that the two effects lead to a clear separation of the cancellation band into two bands after the Daya Bay measurement of ϑ_{13}.

4.2. *Inverted hierarchy of the neutrino masses*

In this case, for the neutrino masses we have

$$m_3 \ll \sqrt{\Delta m_a^2}\,, \qquad m_1 \simeq \sqrt{\Delta m_a^2}\,,$$
$$m_2 \simeq \sqrt{\Delta m_a^2}\left(1+\frac{\Delta m_s^2}{2\Delta m_a^2}\right) \simeq \sqrt{\Delta m_a^2}\,. \tag{60}$$

In the expression of $|m_{\beta\beta}|$, the contribution of the term $m_3 \sin^2\vartheta_{13}$ can be safely neglected. Neglecting also the small contribution of $\sin^2\vartheta_{13}$, from Eq. (48) we find

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m_a^2(1-\sin^2 2\vartheta_{12}\,\sin^2\alpha_{12})}\,. \tag{61}$$

The phase α_{12} is the only unknown parameter in the expression for the effective Majorana mass in the case of a inverted mass hierarchy.

From Eq. (61) we find the following range for $|m_{\beta\beta}|$:

$$\cos 2\vartheta_{12}\sqrt{\Delta m_a^2} \le |m_{\beta\beta}| \le \sqrt{\Delta m_a^2}\,. \tag{62}$$

The upper and lower bounds of this inequality correspond to the case of CP-invariance in the lepton sector. In fact, CP invariance implies that (see Refs. 50, 33 and 57)

$$e^{2i\alpha_{12}} = \eta_2\eta_1^*\,, \tag{63}$$

where $\eta_k = \pm i$ is the CP parity of the Majorana neutrino ν_k. If $\eta_2 = \eta_1$, we have $\alpha_{12} = 0, \pi$ (the upper bound in the inequality (62)). If $\eta_2 = -\eta_1$ we have $\alpha_{12} = \pm\pi/2$ (the lower bound in the inequality (62)).

From the existing neutrino oscillation data, we find the following range for the possible value of the effective Majorana mass:

$$10^{-2} \lesssim |m_{\beta\beta}| \lesssim 5\cdot 10^{-2}\ \text{eV}\,. \tag{64}$$

The anticipated sensitivities to $|m_{\beta\beta}|$ of the future experiments on the search for the $0\nu\beta\beta$-decay are in the range (64) (see Sec. 6). Thus, the future $0\nu\beta\beta$-decay experiments will probe the Majorana nature of neutrinos if a inverted hierarchy of neutrino masses is realized in nature.

4.3. *Quasi-degenerate neutrino mass spectrum*

Neglecting the small contribution of $\sin^2\vartheta_{13}$ in Eq. (48), in the case of a quasi-degenerate neutrino mass spectrum we obtain

$$|m_{\beta\beta}| \simeq m_0\sqrt{1-\sin^2 2\vartheta_{12}\sin^2\alpha_{12}}\,, \tag{65}$$

where m_0 is the unknown absolute mass scale of neutrino masses (see Eq. (56)) and α_{12} is the phase difference between U_{e2} and U_{e1}: $\alpha_{12} = \lambda_2$ in the standard parameterization (5) of the mixing matrix. Thus, in this case $|m_{\beta\beta}|$ depends on two unknown parameters: m_0 and α_{12}.

From Eq. (65), we obtain the following range for the effective Majorana mass:

$$\cos 2\vartheta_{12}\, m_0 \leq |m_{\beta\beta}| \leq m_0 \,. \tag{66}$$

If $0\nu\beta\beta$-decay will be observed and the effective Majorana mass will turn out to be relatively large ($|m_{\beta\beta}| \gg \sqrt{\Delta m_a^2}$), we will have an evidence that neutrinos are Majorana particles and their mass spectrum is quasi-degenerate. In this case, we have

$$|m_{\beta\beta}| \leq m_0 \leq \frac{|m_{\beta\beta}|}{\cos 2\vartheta_{12}} \simeq 2.8|m_{\beta\beta}| \,. \tag{67}$$

Information about the value of the mass scale will be inferred from the data of the future tritium β-decay experiment KATRIN[58,59] and from future cosmological observations. The sensitivity of the KATRIN experiment to the neutrino mass scale is expected to be about 0.2 eV, which the same as the sensitivity to m_β in Eq. (15), since in the quasi-degenerate case $m_\beta \simeq m_0$. Cosmological observations give information on the value of the sum of the neutrino masses $\sum_i m_i \simeq 3m_0$ in the quasi-degenerate case. The existing cosmological data imply the bound $\sum_i m_i \lesssim 0.5$ eV (see Ref. 55). It is expected that future cosmological observations will be sensitive to $\sum_i m_i$ in the range $(6 \times 10^{-3} - 10^{-1})$ eV (see, for example, Ref. 60).

5. Nuclear Matrix Elements

The effective Majorana mass $|m_{\beta\beta}|$ is not a directly measurable quantity. The measurement of the half-life of $0\nu\beta\beta$-decay gives *the product of the effective Majorana mass and the nuclear matrix element* (see Eq. (47)). Hence, in order to determine the effective Majorana mass one must calculate the nuclear matrix elements (NMEs) of $0\nu\beta\beta$-decay, which is a complicated nuclear many-body problem. Five different methods are used at present. In this short review we do not describe these methods and we do not discuss the advantages and disadvantages of each of them. We only present the references to the original papers in Table 2 and the latest results in Fig. 3.

From Fig. 3 we reach the following conclusions:

(1) The LSSM value of each NME is typically smaller than the corresponding one calculated with other approaches. Moreover, the LSSM value of each NME depends weakly on the nucleus, except for the double-magic nucleus ^{48}Ca. If $0\nu\beta\beta$-decay of different nuclei will be observed in future experiments, this

Table 2. Methods of calculation of nuclear matrix elements of $0\nu\beta\beta$-decay.

Method	References
Quasi-particle Random Phase Approximation (QRPA)	61–64
Energy Density Functional method (EDF)	65, 66
Projected Hartree–Fock–Bogoliubov approach (PHFB)	67, 68
Interacting Boson Model-2 (IBM-2)	69–71
Large-Scale Shell Model (LSSM)	72, 73

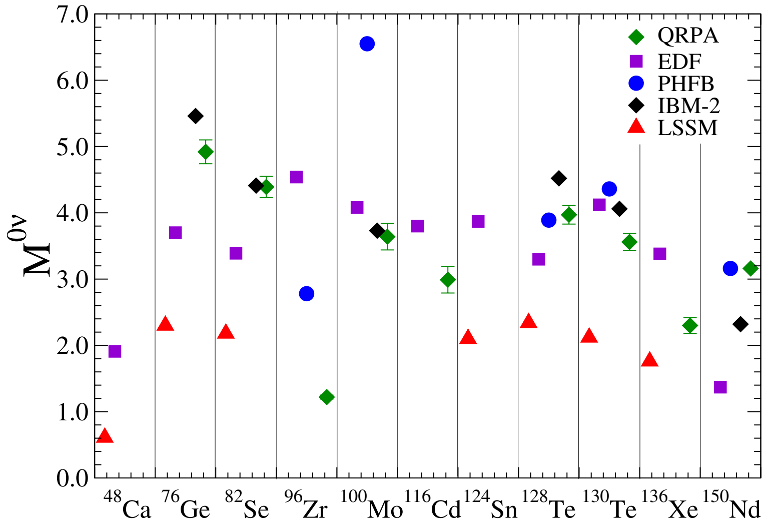

Fig. 3. Values of the NME calculated with the methods in Table 2.[74]

characteristic feature of the LSSM can be checked, because the LSSM predicts the following ratio of half-lives of different nuclei:

$$\frac{T^{0\nu}_{1/2}(Z_1, A_1)}{T^{0\nu}_{1/2}(Z_2, A_2)} \simeq \frac{G^{0\nu}(Q_2, Z_2)}{G^{0\nu}(Q_1, Z_1)}\,. \tag{68}$$

(2) There is a large discrepancy between the values of NMEs calculated with different approaches. The ratios of the maximal and minimal values of each NME are 3.1 (^{48}Ca), 2.4 (^{76}Ge), 2.0 (^{82}Se), 3.7 (^{96}Zr), 1.8 (^{100}Mo), 1.3 (^{116}Cd), 1.8 (^{124}Sn), 1.9 (^{128}Te), 2.1 (^{130}Te), 1.9 (^{136}Xe), 2.3 (^{150}Nd). Therefore, the situation with the calculation of the $0\nu\beta\beta$-decay NMEs is obviously not satisfactory at present. Further efforts and progress are definitely needed.

6. Neutrinoless Double-Beta Decay Experiments

Many experiments searched for neutrinoless double-beta decay without finding an uncontroversial positive evidence. The most stringent lower bounds on the half-lives of the decays of ^{76}Ge, ^{130}Te and ^{100}Mo have been obtained, correspondingly, in the Heidelberg–Moscow,[75] Cuoricino[76] and NEMO3[77,78] experiments.

In the Heidelberg–Moscow experiment[75] Germanium crystals with a 86% enrichment in the $\beta\beta$-decaying isotope ^{76}Ge were used. The total mass of ^{76}Ge was 11 kg, with a low background of 0.11 counts/(kg keV y). After 13 years of running (with a 35.5 kg y exposure) no $\beta\beta$-peak at $Q = 2039$ keV was found. The resulting half-live is

$$T^{0\nu}_{1/2}(^{76}\text{Ge}) > 1.9 \times 10^{25}\ \text{y} \quad (90\%\ \text{CL})\,, \tag{69}$$

which implies that[e]

$$|m_{\beta\beta}| \lesssim (0.22 - 0.64)\ \text{eV}\,. \tag{70}$$

In the cryogenic experiment Cuoricino[76] TeO_2 bolometers were used, with a total mass of 11.34 kg of ^{130}Te. The background was 0.17 counts/(kg keV y). After a 19.75 kg y exposure the following lower bound was obtained:

$$T^{0\nu}_{1/2}(^{130}\text{Te}) > 2.8 \times 10^{24}\ \text{y} \quad (90\%\ \text{CL})\,, \tag{71}$$

which corresponds to

$$|m_{\beta\beta}| \lesssim (0.30\text{–}0.71)\ \text{eV}\,. \tag{72}$$

In the NEMO3 experiment[77,78] the cylindrical source was divided in sectors with enriched ^{100}Mo (6914 g), ^{82}Se (932 g) and other $\beta\beta$-decaying isotopes. The two emitted electrons were detected in drift cells and plastic scintillator. No $0\nu\beta\beta$-decay was observed. The half-life of $0\nu\beta\beta$-decay of ^{100}Mo have been bounded by

$$T^{0\nu}_{1/2}(^{100}\text{Mo}) > 1.1 \times 10^{24}\ \text{y} \quad (90\%\ \text{CL}). \tag{73}$$

The corresponding limit for the effective Majorana mass is

$$|m_{\beta\beta}| \lesssim (0.44\text{–}1.00)\ \text{eV}\,. \tag{74}$$

Several new experiments on the search for $0\nu\beta\beta$-decay of different nuclei are currently running or in preparation. In the following we discuss briefly some of them (for more detailed presentations of future experiments see Refs. 44 and 45).

In the GERDA experiment,[80] started in 2011, 18 kg of enriched germanium crystals (with 86% of the $\beta\beta$-decaying isotope ^{76}Ge) are used. The expected background in the Phase-I of the experiment is 10^{-2} counts/(kg keV y). After one year of running it is expected to reach a sensitivity of $T^{0\nu}_{1/2}(^{76}\text{Ge}) = 2.5 \times 10^{25}$ y, which should allow to check the claim made in Ref. 79.

During the Phase-II of the GERDA experiment (expected to start in 2013), an array of enriched Germanium crystals (with 40 kg of ^{76}Ge) will be cooled and shielded by liquid Argon of very high radiopurity. A low background

[e]Some participants of the Heidelberg–Moscow experiment claimed[79] the observation of $0\nu\beta\beta$-decay of ^{76}Ge with half-life $T^{0\nu}_{1/2}(^{76}\text{Ge}) = (2.23^{+0.44}_{-0.31}) \times 10^{25}$ y (with 51.39 kg y exposure). From this result the authors found $|m_{\beta\beta}| = 0.32 \pm 0.03$ eV. This claim will be checked by the GERDA experiment[80] using the same $0\nu\beta\beta$-decaying nucleus.

(10^{-3} counts/(kg keV y)) is expected. After five years of data taking, in the Phase-II of the experiment a sensitivity of $T^{0\nu}_{1/2}(^{76}\text{Ge}) \simeq 1.9 \cdot 10^{26}$ y is expected. The corresponding sensitivity to the effective Majorana mass is $|m_{\beta\beta}| \simeq (7.3 \cdot 10^{-2}\text{–}2.0 \cdot 10^{-1})$ eV.

In the cryogenic CUORE experiment[81] TeO_2 bolometers are used both as source and as detector. In the Phase-I of the experiment (started in the end of 2011) the target mass is 10.8 kg of ^{130}Te. In the Phase-II (expected to start in 2014) the target mass will be 206 kg of ^{130}Te. The expected background in this phase will be 10^{-2} counts/(kg keV y). After five years of data taking a sensitivity of $T^{0\nu}_{1/2}(^{130}\text{Te}) = 1.6 \cdot 10^{26}$ y will be reached, which corresponds to $|m_{\beta\beta}| \simeq (4.0\text{–}9.4) \cdot 10^{-2}$ eV.

In the KamLAND-Zen experiment,[82] the $0\nu\beta\beta$-decay of ^{136}Xe will be studied. In this experiment enriched Xe (with 91% of the $\beta\beta$-decaying isotope ^{136}Xe) dissolved in liquid scintillator will be placed in a balloon (3.4 m in diameter) at the center of the KamLAND detector. In the first phase of the experiment (started in 2011), the source mass is 364 kg of ^{136}Xe. In the second phase (scheduled for 2013) 910 kg of ^{136}Xe will be utilized. After five years of data taking it will be possible to reach a sensitivity to $|m_{\beta\beta}|$ in the region of the inverted hierarchy ($|m_{\beta\beta}| \simeq 2.5 \cdot 10^{-2}$ eV).

In the running EXO experiment[83] the decay $^{136}\text{Xe} \to {}^{136}\text{Ba} + e^- + e^-$ is searched for. In the first phase of the experiment (EXO-200) the source mass is 50 kg of ^{136}Xe. After two years of data taking a sensitivity $|m_{\beta\beta}| \simeq (8.7 \cdot 10^{-2}\text{–}2.2 \cdot 10^{-1})$ eV is planned to be achieved. The full EXO experiment will consist of about 1 ton of liquid enriched Xe (with 80.6% of the $\beta\beta$-decaying isotope ^{136}Xe). With Ba^+ tagging, a very low background of about 10^{-4} counts/(kg keV y) will be reached. After five years of data taking, it is expected to reach a sensitivity of $T^{0\nu}_{1/2}(^{136}\text{Xe}) \simeq 10^{27}$ y, which corresponds to $|m_{\beta\beta}| \simeq (1.6\text{–}4.0) \cdot 10^{-2}$ eV.

7. Conclusions

If massive neutrinos are Majorana particles, neutrinoless double-beta decay of ^{76}Ge, ^{100}Mo, ^{130}Te, ^{136}Xe and other even–even nuclei is allowed. However, the expected probability of $0\nu\beta\beta$-decay is extremely small, because:

(1) It is a process of second order in the Fermi constant G_F.
(2) Since the Hamiltonian of weak interactions conserves helicity, the amplitude of $0\nu\beta\beta$-decay is proportional to the very small factor

$$\frac{m_{\beta\beta}}{\bar{q}^2}, \tag{75}$$

which comes from the neutrino propagator. Here $m_{\beta\beta} = \sum_i U^2_{ei} m_i$ is the effective Majorana mass ($\lesssim 1$ eV) and $\bar{q}$ is the average neutrino momentum (~ 100 MeV).

The expected half-lives of $0\nu\beta\beta$-decays depend on the decaying nucleus and are typically larger than 10^{24}–10^{25} years. Therefore, the observation of this rare process is a real challenge.

The effective Majorana mass (and consequently the matrix element of the process) depends on the character of the neutrino mass spectrum.

In the case of a quasi-degenerate spectrum, the expected value of $m_{\beta\beta}$ is relatively large. This case is partly excluded by the data of the performed $0\nu\beta\beta$-decay experiments and by cosmological data (see Fig. 2). It will be further explored by GERDA, KamLAND-Zen, EXO, CUORE and other experiments.

In order to reach the region of the inverted neutrino mass hierarchy, with $10^{-2} \lesssim |m_{\beta\beta}| \lesssim 5 \cdot 10^{-2}$ eV, the construction of large detectors ($\sim$ 1 ton) and about five years of data taking will be required.

We considered here the $0\nu\beta\beta$-decay induced by the standard mechanism of exchange of light Majorana neutrinos between n-p-e^- vertices. From neutrino oscillation data it follows that if neutrino with definite masses are Majorana particles this decay mechanism is realized if there is no cancellation of the different mass contributions (as shown in Fig. 2, cancellations can happen in the normal scheme).

As discussed in Sec. 2, the neutrino mass mechanism of $0\nu\beta\beta$-decay is predicted by the standard seesaw mechanism.[34–37] However, additional sources of violation of the total lepton number L are possible (see Ref. 84 and references therein). If L is violated at the TeV scale these additional mechanisms could give contributions to the matrix elements of the $0\nu\beta\beta$-decay comparable with the contribution of the light Majorana neutrino mass mechanism.

Let us consider as an example the violation of L due to R-parity violating interactions of SM and SUSY particles. In this case, $0\nu\beta\beta$-decay is induced by the exchange of a heavy Majorana SUSY neutralino. The product of n-p-e^- vertices is given by the factor

$$\left(\frac{G_{\mathrm{F}}}{\sqrt{2}}\right)^2 \left(\frac{m_W^2}{\Lambda^2}\right)^2 \frac{1}{\Lambda}\,, \tag{76}$$

where Λ characterizes the scale of the masses of SUSY particles and m_W is the mass of the W-boson. The factor (76) must be compared with the corresponding factor

$$\left(\frac{G_{\mathrm{F}}}{\sqrt{2}}\right)^2 \left(\frac{m_{\beta\beta}}{\bar{q}^2}\right), \tag{77}$$

which appears in the case of the Majorana neutrino mass mechanism. Taking into account that $\bar{q} \simeq 100$ MeV and assuming that $|m_{\beta\beta}| \simeq 10^{-1}$ eV, we come to the conclusion that Eqs. (76) and (77) are comparable if Λ is of the order of a few TeV.

If the $0\nu\beta\beta$-decay of different nuclei will be observed in future experiments, it will be possible to probe the presence of different mechanisms which can generate the process.

Finally, let us emphasize that the search for $0\nu\beta\beta$-decay is a powerful practical way to solve one of the most fundamental problem of modern neutrino physics: *are neutrinos with definite masses ν_i truly neutral Majorana particles or are they Dirac particles possessing a conserved total lepton number?*

Acknowledgments

We are grateful to F. Šimkovic who kindly provided us with Fig. 3. We are also thankful to him for useful discussions.

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