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Violation of time-reversal invariance and CPLEAR measurements

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Abstract

Motivated by the recent CPLEAR measurement on the time-reversal non-invariance, we review the situation concerning the experimental measurements of charge conjugation, parity violation and time reversibility, in systems with non-Hermitean Hamiltonians. This includes in particular neutral meson systems, like $K^0 - \overline{K}^0$, $D^0 - \overline{D}^0$ and $B^0 - \overline{B}^0$. We discuss the formalism that describes particle-antiparticle mixing and time evolution of states, paying particular emphasis to the orthogonality conditions of incoming and outgoing states. As a result, we confirm that the CPLEAR experiment makes a direct measurement of violation of time-reversal without any assumption of unitarity and *CPT*-violation. The asymmetry which signifies *T*-violation, is found to be independent of time and decay processes.

1. Introduction

Recently, the CPLEAR experiment at CERN, reported the first direct observation of time-reversal violation in the neutral kaon system [1]. This observation is made by comparing the probabilities of a $\overline{K}^0$ state transforming into a K^0 and vice-versa. CPLEAR produces initial neutral kaons with defined strangeness from proton-antiproton annihilations at rest, via the reactions

$$p\bar{p} \to \begin{cases} K^-\pi^+K^0 \\ K^+\pi^-\overline{K}^0 , \end{cases}$$

and tags the neutral kaon strangeness at the production time by the charge of the accompanying charged kaon. Since weak interactions do not conserve strangeness, the K^0 and $\overline{K}^0$ may subsequently transform into each other via oscillations with $\Delta S = 2$. The final strangeness of the neutral kaon is then tagged through semi-leptonic decays. In this way,

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among other quantities, CPLEAR also measured the asymmetry [1,2]

$$A_T^{\exp} = \frac{R(\bar{K}^0(t=0) \to e^+\pi^-\nu(t=\tau)) - R(K^0(t=0) \to e^-\pi^+\bar{\nu}(t=\tau))}{R(\bar{K}^0(t=0) \to e^+\pi^-\nu(t=\tau)) + R(K^0(t=0) \to e^-\pi^+\bar{\nu}(t=\tau))}, \tag{1}$$

which parametrizes the difference of the probability that an initial $\bar{K}^0(t_i)$ oscillates to a final $K^0(t_f)$, from the probability that an initial $K^0(t_i)$ oscillates to a final $\bar{K}^0(t_f)$. The average value of $A_T^{\exp}$ was found over the time interval from $1\tau_S$ to $20\tau_S$ (where τ_S is the lifetime of the short-lived kaon), to be different than zero by 4σ and this has been interpreted by CPLEAR as the first direct measurement of time-reversal non-invariance.

However, doubts have been expressed concerning the interpretation of the CPLEAR result in providing such a direct evidence for *T*-violation. The basic argument is that decay processes enter in the observables, making *CP*-violation manifest. The observed effect is then attributed to these irreversible processes, rather than *T*-violation. It is also argued that this is only a direct effect of the decaying states being non-orthogonal.

The aim of this work is to clarify these points. In order to do so, we are going to re-discuss the formalism that describes the particle-antiparticle mixing and time evolution of states in the kaon system. Since the Hamiltonian H of the system is non-Hermitean, the various masses, widths and eigenstates have to be found by using two matrices V_L and V_R such that $V_L H V_R$ is diagonal. The form of these matrices is found by left- and right- independent diagonalizations, while the physical states are defined by the V_L^{-1} and V_R^{-1} ''rotations'' of the left- and right-states. This is equivalent to identifying the form of the matrices and the eigenstates, by looking consistently at the correct orthogonality conditions for the outgoing and and incoming states. This analysis is done in Section 2, where we describe the states in the vector space of the system, its dual, as well as the dual complex space. In Section 3, we are going to show that the theoretical asymmetry which arises directly from the *definition of T-violation*, is independent of time and decay processes. In Section 4, we point out that this is also true for the experimental asymmetry that CPLEAR uses, which differs from the theoretical one due to the appearance of the semi-leptonic decays in the process. In the same section, we show that since the experiment uses a specific search-channel, rather than summing over all possible modes, *no unitarity or CPT-invariance* arguments enter in the analysis. Finally, in Section 5 we present a summary of the basic points and conclude that the CPLEAR experiment indeed makes a direct measurement of *T*-violation.

2. Definition of states in the incoming $\mathcal{H}^{\text{in}}$ and outgoing $\mathcal{H}^{\text{out}}$ dual spaces

In this section we are going to use the standard treatment of the theory of non-Hermitean Hamiltonians. This, together with an application to the $K-\bar{K}$-system can be found in Ref. [3]. We denote by $\mathcal{H}^{\text{in}}$ and $\mathcal{H}^{\text{out}}$ the Hilbert space of incoming and outgoing (dual) states, respectively.

$$\mathcal{H}^{\text{in}} \equiv \left\{ |\Psi_I^{\text{in}}\rangle, I = 1,2,\dots,n \right\},$$
$$\mathcal{H}^{\text{out}} \equiv \left\{ \langle \Psi_I^{\text{out}}|, \ I = 1,2,\dots,n \right\}, \tag{2}$$

n is the dimension of the space and $|\Psi_I^{\text{in}}\rangle$ and $\langle\Psi_I^{\text{out}}|$ are the *right*- and *left*-eigenstates [5] of the effective Hamiltonian H:

$$H|\Psi_I^{\text{in}}\rangle = \lambda_I |\Psi_I^{\text{in}}\rangle, \quad \langle\Psi_I^{\text{out}}| H = \langle\Psi_I^{\text{out}}| \lambda_I . \tag{3}$$

In this basis, the effective Hamiltonian is diagonal and can be expressed in the following form in terms of the incoming and outgoing states:

$$H = \sum |\Psi_I^{\text{in}}\rangle \lambda_I \langle\Psi_I^{\text{out}}|, \quad \text{with} \quad \langle\Psi_I^{\text{out}}|\Psi_J^{\text{in}}\rangle = \delta_{IJ}, \tag{4}$$

where the unity operator **1** takes the usual form:

$$\mathbf{1} = \sum |\Psi_J^{\text{in}}\rangle\langle\Psi_I^{\text{out}}|. \tag{5}$$

Up to this point, we *do not assume* that H is Hermitean; $H \neq H^\dagger$. This implies that the conjugate

[5] Technically, we assume that the Hamiltonian H is an $n \times n$ matrix with n well-defined left- and right-eigenvectors, to avoid some pathological cases that are irrelevant in the $K^0-\bar{K}^0$ system [4]. .

states $\langle \Psi_I^{\rm out}|^\dagger$ and $|\Psi_I^{\rm in}\rangle^\dagger$ are not isomorphic to their duals:

$$|\Psi_I^{\rm out}\rangle \equiv \langle \Psi_I^{\rm out}|^\dagger \neq |\Psi_I^{\rm in}\rangle,$$
$$\langle \Psi_I^{\rm in}| \equiv |\Psi_I^{\rm in}\rangle^\dagger \neq \langle \Psi_I^{\rm out}|. \quad (6)$$

The vectors, $|\Psi_I^{\rm out}\rangle$ and $\langle \Psi_I^{\rm in}|$ are eigenstates of the $H^\dagger$ operator but they *are not eigenstates* of H:

$$H^\dagger |\Psi_I^{\rm out}\rangle = \lambda_I^* |\Psi_I^{\rm out}\rangle, \quad \langle \Psi_I^{\rm in}| H^\dagger = \langle \Psi_I^{\rm in}| \lambda_I^* . \quad (7)$$

Only if the effective Hamiltonian is Hermitean, (i.e. $H = H^\dagger$), the conjugate outgoing states become isomorphic to the incoming ones, $|\Psi_I^{\rm out}\rangle = |\Psi_I^{\rm in}\rangle$; in this case the eigenvalues $\lambda_I = \lambda_I^*$ are real.

When $H \neq H^\dagger$, the time evolution of the incoming and outgoing states $|\Psi_I^{\rm in}(t_i)\rangle$ and $|\Psi_I^{\rm out}(t_f)\rangle$ are obtained from $|\Psi_I^{\rm in}\rangle$ and $|\Psi_I^{\rm out}\rangle$, using the evolution operators e^{-iHt_i} and $e^{-iH^\dagger t_f}$ respectively:

$$|\Psi_I^{\rm in}(t_i)\rangle = e^{-iHt_i}|\Psi_I^{\rm in}\rangle,$$
$$|\Psi_I^{\rm out}(t_f)\rangle = e^{-iH^\dagger t_f}|\Psi_I^{\rm out}\rangle. \quad (8)$$

From the above equations, follows the evolution of the conjugate states:

$$\langle \Psi_I^{\rm in}(t_i)| = \langle \Psi_I^{\rm in}| e^{iH^\dagger t_i},$$
$$\langle \Psi_I^{\rm out}(t_f)| = \langle \Psi_I^{\rm out}| e^{iHt_f}. \quad (9)$$

In view of our later discussion, it is important to stress here that the inner products among incoming and outgoing states *do not obey* the usual orthogonality conditions. Indeed,

$$\langle \Psi_I^{\rm out}|\Psi_J^{\rm out}\rangle \neq \delta_{IJ} \quad \text{and} \quad \langle \Psi_I^{\rm in}|\Psi_J^{\rm in}\rangle \neq \delta_{IJ}. \quad (10)$$

On the other hand, the physical incoming and outgoing eigenstates obey *at all times* the orthogonality conditions

$$\langle \Psi_I^{\rm out}(t_f)|\Psi_J^{\rm in}(t_i)\rangle = \langle \Psi_I^{\rm out}|e^{-iH\Delta t}|\Psi_J^{\rm in}\rangle = e^{-i\lambda_I \Delta t}\delta_{IJ}. \quad (11)$$

We now proceed to discuss particle-antiparticle mixing in the neutral kaon system.

3. Particle-antiparticle mixing in the neutral kaon system

The $K^0, \bar{K}^0$ states are produced under strong interactions and are strangeness eigenstates. Moreover, they obey the relations:

$$CP\,|K_0^{\rm in}\rangle = |\bar{K}_0^{\rm in}\rangle, \quad T\,|K_0^{\rm in}\rangle = \langle K_0^{\rm out}|,$$
$$CPT\,|K_0^{\rm in}\rangle = \langle \bar{K}_0^{\rm out}|. \quad (12)$$

These states are admixtures of the physical *incoming* ($|K_S^{\rm in}\rangle$ and $|K_L^{\rm in}\rangle$) and *outgoing* ($\langle K_S^{\rm out}|$ and $\langle K_L^{\rm out}|$) states of the full Hamiltonian and obey the following orthogonality conditions:

$$\langle K_L^{\rm out}|K_S^{\rm in}\rangle = 0, \quad \langle K_S^{\rm out}|K_L^{\rm in}\rangle = 0,$$
$$\langle K_S^{\rm out}|K_S^{\rm in}\rangle = 1, \quad \langle K_L^{\rm out}|K_L^{\rm in}\rangle = 1. \quad (13)$$

The physical states, are the left- and right-eigenvectors of the effective Hamiltonian of the system, $H \equiv M - i\Gamma/2$:

$$H\,|K_L^{\rm in}\rangle = \lambda_L\,|K_L^{\rm in}\rangle, \quad H\,|K_S^{\rm in}\rangle = \lambda_S\,|K_S^{\rm in}\rangle,$$
$$\langle K_L^{\rm out}|\,H = \langle K_L^{\rm out}|\,\lambda_L, \quad \langle K_S^{\rm out}|\,H = \langle K_S^{\rm out}|\,\lambda_S. \quad (14)$$

Since H is not Hermitean, this implies in general that the incoming and outgoing eigenvectors in the K^0, $\bar{K}^0$ base *are not related* simply by complex conjugation.

Without loss of generality, we can express the physical incoming states in terms of $|K_0^{\rm in}\rangle$ and $|\bar{K}_0^{\rm in}\rangle$ as:

$$|K_S^{\rm in}\rangle = \frac{1}{N_S}\Big((1+\alpha)|K_0^{\rm in}\rangle + (1-\alpha)\,|\bar{K}_0^{\rm in}\rangle\Big),$$
$$|K_L^{\rm in}\rangle = \frac{1}{N_L}\Big((1+\beta)|K_0^{\rm in}\rangle - (1-\beta)\,|\bar{K}_0^{\rm in}\rangle\Big), \quad (15)$$

where α and β are complex variables associated with CP,T and CPT-violation, and N_L, N_S are normalization factors to be discussed below. Similar relations exist for the dual outgoing states:

$$\langle K_S^{\rm out}| = \frac{1}{\tilde{N}_S}\left((1+\tilde{\alpha})\langle K_0^{\rm out}| + (1-\tilde{\alpha})\langle \bar{K}_0^{\rm out}|\right),$$
$$\langle K_L^{\rm out}| = \frac{1}{\tilde{N}_L}\left((1+\tilde{\beta})\langle K_0^{\rm out}| - (1-\tilde{\beta})\langle \bar{K}_0^{\rm out}|\right). \tag{16}$$

The parameters (α,β) and $(\tilde{\alpha},\tilde{\beta})$ that are associated with the incoming and outgoing states respectively, are not independent but are related through the orthogonality conditions (Eqs. (13)) valid for the physical states:

$$\langle K_L^{\rm out}|K_S^{\rm in}\rangle = 0 \Rightarrow \tilde{\beta} = -\alpha\,,$$
$$\langle K_S^{\rm out}|K_L^{\rm in}\rangle = 0 \Rightarrow \tilde{\alpha} = -\beta\,,$$
$$\langle K_S^{\rm out}|K_S^{\rm in}\rangle = 1 \Rightarrow N_S\tilde{N}_S = 2(1-\alpha\beta)\,,$$
$$\langle K_L^{\rm out}|K_L^{\rm in}\rangle = 1 \Rightarrow N_L\tilde{N}_L = 2(1-\alpha\beta)\,. \tag{17}$$

The above relations indicate that, while the normalizations $\tilde{N}_{S,L}$ can be expressed in terms of $N_{S,L}$, the latter remain unspecified. This ambiguity however will not affect any measurable quantity. Thus we can always choose

$$N \equiv N_S = \tilde{N}_S = N_L = \tilde{N}_L = \sqrt{2(1-\alpha\beta)}\,. \tag{18}$$

Let us write down for completeness the inverse transformations that express the $K^0,\bar{K}^0$ states in terms of K_S and K_L:

$$|K_0^{\rm in}\rangle = \frac{1}{N}\left((1-\beta)|K_S^{\rm in}\rangle + (1-\alpha)|K_L^{\rm in}\rangle\right),$$
$$|\bar{K}_0^{\rm in}\rangle = \frac{1}{N}\left((1+\beta)|K_S^{\rm in}\rangle - (1+\alpha)|K_L^{\rm in}\rangle\right), \tag{19}$$

and

$$\langle K_0^{\rm out}| = \frac{1}{N}\left((1+\alpha)\langle K_S^{\rm out}| + (1+\beta)\langle K_L^{\rm out}|\right),$$
$$\langle \bar{K}_0^{\rm out}| = \frac{1}{N}\left((1-\alpha)\langle K_S^{\rm out}| - (1-\beta)\langle K_L^{\rm out}|\right). \tag{20}$$

In the basis of the states K_L,K_S, H can be expressed in terms of a diagonal 2×2 matrix

$$H = |K_S^{\rm in}\rangle\lambda_S\langle K_S^{\rm out}| + |K_L^{\rm in}\rangle\lambda_L\langle K_L^{\rm out}|, \tag{21}$$

while in the basis of $K^0,\bar{K}^0$, H takes the following form:

$$H_{ij} = \frac{1}{2}\begin{pmatrix} (\lambda_L+\lambda_S) - \Delta\lambda\frac{(\alpha-\beta)}{1-\alpha\beta} & \Delta\lambda\frac{(1+\alpha\beta)}{1-\alpha\beta} + \Delta\lambda\frac{\alpha+\beta}{1-\alpha\beta} \\ \Delta\lambda\frac{(1+\alpha\beta)}{1-\alpha\beta} - \Delta\lambda\frac{\alpha+\beta}{1-\alpha\beta} & (\lambda_L+\lambda_S) + \Delta\lambda\frac{\alpha-\beta}{1-\alpha\beta} \end{pmatrix}. \tag{22}$$

Here,

$$\Delta\lambda = \lambda_S - \lambda_L\,, \quad \lambda_L = m_L - i\frac{\Gamma_L}{2}\,, \quad \lambda_S = m_S - i\frac{\Gamma_S}{2}\,,$$

where m_S,m_L are the K_S,K_L masses and Γ_S,Γ_L, the K_S,K_L widths. From Eq. (22), we can identify the *T*-, *CP*- and *CPT*- violating parameters. Indeed:

- Under <u>*T*–transformations</u>,

  $$\langle K_0^{\rm out}|H|\bar{K}_0^{\rm in}\rangle \leftrightarrow \langle \bar{K}_0^{\rm out}|H|K_0^{\rm in}\rangle,$$

  thus, the off-diagonal elements of H are interchanged. This indicates that the parameter $\epsilon \equiv (\alpha+\beta)/2$, which is related to the difference of the off-diagonal elements of H, measures the magnitude of the *T*-violation [6].

  $$\frac{2}{N^2}\epsilon = \frac{\langle K_0^{\rm out}|H|\bar{K}_0^{\rm in}\rangle - \langle \bar{K}_0^{\rm out}|H|K_0^{\rm in}\rangle}{2\,\Delta\lambda}\,. \tag{23}$$

- Under <u>*CPT*–transformations</u>,

  $$\langle K_0^{\rm out}|H|K_0^{\rm in}\rangle \leftrightarrow \langle \bar{K}_0^{\rm out}|H|\bar{K}_0^{\rm in}\rangle,$$

  and therefore, the parameter $\delta \equiv (\alpha-\beta)/2$, related to the difference of the diagonal elements of H, measures the magnitude of *CPT*-violation.

  $$\frac{2}{N^2}\delta = \frac{\langle \bar{K}_0^{\rm out}|H|\bar{K}_0^{\rm in}\rangle - \langle K_0^{\rm out}|H|K_0^{\rm in}\rangle}{2\,\Delta\lambda}\,. \tag{24}$$

- Under <u>*CP*–transformation</u>,

  $$\langle K_0^{\rm out}|H|K_0^{\rm in}\rangle \leftrightarrow \langle \bar{K}_0^{\rm out}|H|\bar{K}_0^{\rm in}\rangle,$$

  and simultaneously

  $$\langle K_0^{\rm out}|H|\bar{K}_0^{\rm in}\rangle \leftrightarrow \langle \bar{K}_0^{\rm out}|H|K_0^{\rm in}\rangle,$$

  thus, *both* the diagonal and the off-diagonal elements of H are interchanged. Then, the parame-

[6] $2/N^2 \approx 1$, in the linear approximation.

ters $\alpha = \epsilon + \delta$ and $\beta = \epsilon - \delta$, usually denoted as ϵ_S and ϵ_L, are the ones which measure the magnitude of *CP*-violation in the decays of K_S and K_L respectively.

4. Direct measurement testing time-reversibility

The meaning of classical time-reversal invariance is unambiguous. A system at a final classical configuration retraces its way back to some initial configuration by reversing the velocities. As a result of time-reversal invariance, initial and final quantum mechanical states are interchanged with identical positions and opposite velocities:

$$T\left[\langle \Psi^{\text{out}}(t_f)|\Phi^{\text{in}}(t_i)\rangle\right] = \langle \Phi^{\text{out}}(t_f)|\Psi^{\text{in}}(t_i)\rangle . \tag{25}$$

In order to test time reversibility, one has to compare the magnitude of the probability $|\langle \Psi^{\text{out}}(t_f)|\Phi^{\text{in}}(t_i)\rangle|^2$ with that of the time-reversed process $|\langle \Phi^{\text{out}}(t_f)|\Psi^{\text{in}}(t_i)\rangle|^2$. Any possible difference in the two probabilities will signal deviations of time-reversibility. In that case, the process is not equivalent to its time reversed one, resulting in time-reversal violation. In the neutral kaon system, at a given time t_i one has an initial strangeness eigenstate, such that $|K_0^{\text{in}}(t_i)\rangle = |\Psi^{\text{in}}(t_i)\rangle$. At some later time t_f, one finds a final strangeness eigenstate $\langle \overline{K}_0^{\text{out}}(t_f)| = \langle \Phi^{\text{out}}(t_f)|$. According to time-reversibility, we may conclude that the above process should have the same probability with the reversed one, namely, an initial $|\overline{K}_0^{\text{in}}(t_i)\rangle$ to be transformed into a final $\langle K_0^{\text{out}}(t_f)|$. Then, for the kaon system we can write for the case of time-reversal invariance:

$$|\langle \overline{K}_0^{\text{out}}(t_f)|K_0^{\text{in}}(t_i)\rangle|^2 = |\langle K_0^{\text{out}}(t_f)|\overline{K}_0^{\text{in}}(t_i)\rangle|^2 . \tag{26}$$

Any deviation from the above equality will definitely signal time-reversal violation. The comparison of the probabilities of a $\overline{K}^0$ transforming into K^0, and K^0 transforming into $\overline{K}^0$ can demonstrate a departure from time-reversal invariance. More explicitly, such a departure is manifest in the asymmetry

$$A_T = \frac{P_{\overline{K}K}(\Delta t) - P_{K\overline{K}}(\Delta t)}{P_{\overline{K}K}(\Delta t) + P_{K\overline{K}}(\Delta t)} = \frac{|\langle K_0^{\text{out}}(t_f)|\overline{K}_0^{\text{in}}(t_i)\rangle|^2 - |\langle \overline{K}_0^{\text{out}}(t_f)|K_0^{\text{in}}(t_i)\rangle|^2}{|\langle K_0^{\text{out}}(t_f)|\overline{K}_0^{\text{in}}(t_i)\rangle|^2 + |\langle \overline{K}_0^{\text{out}}(t_f)|K_0^{\text{in}}(t_i)\rangle|^2} , \tag{27}$$

known in the literature as the Kabir asymmetry [5].

The time evolution from t_i to t_f is induced by the effective Hamiltonian H:

$$\begin{aligned} A_{K_0 \to \overline{K}_0} &= \langle \overline{K}_0^{\text{out}}(t_f)|K_0^{\text{in}}(t_i)\rangle \\ &= \langle \overline{K}_0^{\text{out}}|e^{-iH\Delta t}|K_0^{\text{in}}\rangle , \\ A_{\overline{K}_0 \to K_0} &= \langle K_0^{\text{out}}(t_f)|\overline{K}_0^{\text{in}}(t_i)\rangle \\ &= \langle K_0^{\text{out}}|e^{-iH\Delta t}|\overline{K}_0^{\text{in}}\rangle . \end{aligned} \tag{28}$$

Inserting the unity operator

$$\mathbf{1} = |K_L^{\text{in}}\rangle\langle K_L^{\text{out}}| + |K_S^{\text{in}}\rangle\langle K_S^{\text{out}}| , \tag{29}$$

to the right of the evolution operator $e^{-iH\Delta t}$ and using the fact that $K_{L,S}$ are Hamiltonian eigenstates, we obtain:

$$\begin{aligned} A_{K_0 \to \overline{K}_0} &= \langle \overline{K}_0^{\text{out}}|K_L^{\text{in}}\rangle\langle K_L^{\text{out}}|K_0^{\text{in}}\rangle e^{-i\lambda_L \Delta t} \\ &\quad + \langle \overline{K}_0^{\text{out}}|K_S^{\text{in}}\rangle\langle K_S^{\text{out}}|K_0^{\text{in}}\rangle e^{-i\lambda_S \Delta t} \\ &= \frac{1}{N^2}(1-\alpha)(1-\beta) \\ &\quad \times \left(e^{-i\lambda_S \Delta t} - e^{-i\lambda_L \Delta t}\right) , \end{aligned} \tag{30}$$

and

$$\begin{aligned} A_{\overline{K}_0 \to K_0} &= \langle K_0^{\text{out}}|K_L^{\text{in}}\rangle\langle K_L^{\text{out}}|\overline{K}_0^{\text{in}}\rangle e^{-i\lambda_L \Delta t} \\ &\quad + \langle K_0^{\text{out}}|K_S^{\text{in}}\rangle\langle K_S^{\text{out}}|\overline{K}_0^{\text{in}}\rangle e^{-i\lambda_S \Delta t} \\ &= \frac{1}{N^2}(1+\alpha)(1+\beta) \\ &\quad \times \left(e^{-i\lambda_S \Delta t} - e^{-i\lambda_L \Delta t}\right) . \end{aligned} \tag{31}$$

We see that the time-dependent factor $g(\Delta t) \equiv (e^{-i\lambda_S \Delta t} - e^{-i\lambda_L \Delta t})$, whose absolute value square is given by

$$\begin{aligned} |g(\Delta t)|^2 &= e^{-\Gamma_S \Delta t} + e^{-\Gamma_L \Delta t} \\ &\quad - 2\cos(m_L - m_S)\,\Delta t\, e^{-\frac{\Gamma_S + \Gamma_L}{2}\Delta t} , \end{aligned} \tag{32}$$

is common in both amplitudes and therefore will cancel in the asymmetry A_T, which becomes *time-independent* [5]. Thus

$$A_T = \frac{|(1+\alpha)(1+\beta)|^2 - |(1-\alpha)(1-\beta)|^2}{|(1+\alpha)(1+\beta)|^2 + |(1-\alpha)(1-\beta)|^2}, \tag{33}$$

We note therefore that a non-zero value for A_T signals a direct measurement of T-violation without any assumption about CPT invariance. Making the substitutions $\alpha = \epsilon + \delta$ and $\beta = \epsilon - \delta$, and keeping only linear terms, one finds that

$$A_T \approx 4\mathrm{Re}[\epsilon]. \tag{34}$$

To make clear the misunderstandings in the literature, [6–14] (with the exception of Ref. [15,16]) we need to introduce the adjoint outgoing states:

$$\langle K_S^{\mathrm{in}}| = \frac{1}{N^*}\left((1+\alpha^*)\langle K_0^{\mathrm{in}}| + (1-\alpha^*)\langle \bar{K}_0^{\mathrm{in}}|\right),$$
$$\langle K_L^{\mathrm{in}}| = \frac{1}{N^*}\left((1+\beta^*)\langle K_0^{\mathrm{in}}| - (1-\beta^*)\langle \bar{K}_0^{\mathrm{in}}|\right). \tag{35}$$

Notice that the adjoint states $\langle K_S^{\mathrm{in}}|$ and $\langle K_L^{\mathrm{in}}|$, are not orthogonal to $|K_S^{\mathrm{in}}\rangle$ and $|K_L^{\mathrm{in}}\rangle$:

$$\langle K_S^{\mathrm{in}}|K_S^{\mathrm{in}}\rangle = \frac{1+|\alpha|^2}{|1-\alpha\beta|}, \quad \langle K_L^{\mathrm{in}}|K_L^{\mathrm{in}}\rangle = \frac{1+|\beta|^2}{|1-\alpha\beta|},$$
$$\langle K_S^{\mathrm{in}}|K_L^{\mathrm{in}}\rangle = \frac{\alpha^*+\beta}{|1-\alpha\beta|}, \quad \langle K_L^{\mathrm{in}}|K_S^{\mathrm{in}}\rangle = \frac{\alpha+\beta^*}{|1-\alpha\beta|},$$
$$\rightarrow \quad \langle K_S^{\mathrm{in}}|K_L^{\mathrm{in}}\rangle + \langle K_L^{\mathrm{in}}|K_S^{\mathrm{in}}\rangle$$
$$= \frac{2\mathrm{Re}[(\alpha+\beta)]}{|1-\alpha\beta|} = \frac{4\mathrm{Re}[\epsilon]}{|1-\alpha\beta|}. \tag{36}$$

In linear order in ϵ and δ, the approximate equality

$$A_T \approx \langle K_S^{\mathrm{in}}|K_L^{\mathrm{in}}\rangle + \langle K_L^{\mathrm{in}}|K_S^{\mathrm{in}}\rangle \approx 4\mathrm{Re}[\epsilon], \tag{37}$$

holds. This relation may result in the misleading conclusion that $A_T \neq 0$ is not associated with T-violation, but rather with the non-orthogonality of the physical incoming states K_L^{in} and K_S^{in} states, and with the violation of CP. However, as we already stressed, (i) the relevant physical states $\langle K_L^{\mathrm{out}}|$ and $|K_S^{\mathrm{in}}\rangle$ are *always orthogonal* (see Eq. (13)) and (ii) A_T is *by definition* the magnitude of T-violation, without any assumption about the validity of CPT or even unitarity.

To better illustrate this point, let us imagine that the CP-violating part α of K_S is zero. In this case $\epsilon = -\delta$, so that T is violated together with CPT, with CP invariance in the K_S decays. Besides, if CPT is assumed, then $\delta = 0$ and $\epsilon = \alpha = \beta$. In that case, clearly, T-violation is identical to CP-violation.

5. The CPLEAR measurement

Up to now, we described the behaviour of the theoretical asymmetry that stems directly from the definition of T-reversal. However, as we mentioned in the introduction, CPLEAR tags the strangeness of the final states through the semi-leptonic decays

$$K^0 \to e^+\pi^-\nu, \quad \bar{K}^0 \to e^-\pi^+\bar{\nu},$$
$$K^0 \to e^-\pi^+\bar{\nu}, \quad \bar{K}^0 \to e^+\pi^-\nu. \tag{38}$$

Among them, the first two are characterized by $\Delta S = \Delta Q$ while the other two are characterized by $\Delta S = -\Delta Q$ and would therefore indicate either (i) explicit violations of the $\Delta S = \Delta Q$ rule, or (ii) oscillations between K^0 and $\bar{K}^0$ that even if $\Delta S = \Delta Q$ holds, would lead at a final state similar to (i) (with the ‘‘wrong-sign’’ leptons).

The experimental asymmetry of Eq. (1) is:

$$A_T^{\mathrm{exp}} = \frac{\bar{R}_+(\Delta t) - R_-(\Delta t)}{\bar{R}_+(\Delta t) + R_-(\Delta t)}, \tag{39}$$

where

$$\bar{R}_+(\Delta t) = |\langle e^+\pi^-\nu(t_f)|\bar{K}_0^{\mathrm{in}}(t_i)\rangle + \langle e^+\pi^-\nu(t_f)|K_0^{\mathrm{in}}(t_f)\rangle \times \langle K_0^{\mathrm{out}}(t_f)|\bar{K}_0^{\mathrm{in}}(t_i)\rangle|^2, \tag{40}$$

$$R_-(\Delta t) = |\langle e^-\pi^+\bar{\nu}(t_f)|K_0^{\mathrm{in}}(t_i)\rangle + \langle e^-\pi^+\bar{\nu}(t_f)|\bar{K}_0^{\mathrm{in}}(t_f)\rangle \times \langle \bar{K}_0^{\mathrm{out}}(t_f)|K_0^{\mathrm{in}}(t_i)\rangle|^2. \tag{41}$$

In the above expressions, the first term in each sum stands for $\Delta S = -\Delta Q$ contributions, while the second contains the kaon oscillations multiplied by the matrix element for semi-leptonic decays through $\Delta S = \Delta Q$. The experimental asymmetry $A_T^{\exp}$ therefore, besides ϵ, also contains the parameters x_- and y, where x_- measures $\Delta S = -\Delta Q$ in $\mathscr{C}PT$-violating amplitudes, while y stands for $\mathscr{C}PT$-violation in the decays when the $\Delta S = \Delta Q$ rule holds [11,14]. Although the $\Delta S = \Delta Q$ rule is expected from the Standard Model to be valid up to order 10^{-14}, the experimental limit before CPLEAR was much larger [17]. For this reason, the $\Delta S = -\Delta Q$ contributions have been retained in the CPLEAR analysis [1,2].

Even in the absence of $\Delta S = -\Delta Q$ contributions, the expression (37) for the time-reversal asymmetry is modified, since the squared matrix elements

$$|\langle e^+ \pi^- \nu(t_f) | K_0^{\text{in}}(t_f) \rangle|^2 \equiv |a|^2 \, |1-y|^2 ,$$
$$|\langle e^- \pi^+ \bar{\nu}(t_f) | \bar{K}_0^{\text{in}}(t_f) \rangle|^2 \equiv |a|^2 \, |1+y|^2 , \qquad (42)$$

enter in the calculation. Even if y is included, the time-independence of the asymmetry still holds (which is no longer the case if the $\Delta S = -\Delta Q$ contributions are also retained). However, y does enter in the asymmetry calculation:

$$A_T = \frac{|(1+\alpha)(1+\beta)|^2 |1-y|^2 - |(1-\alpha)(1-\beta)|^2 |1+y|^2}{|(1+\alpha)(1+\beta)|^2 |1-y|^2 + |(1-\alpha)(1-\beta)|^2 |1+y|^2} . \qquad (43)$$

In particular for the linear approximation, in the absence of $\Delta S = -\Delta Q$ contributions, one finds that

$$A_T^{\exp} \approx 4\mathrm{Re}[\epsilon] - 2\mathrm{Re}[y], \qquad (44)$$

which becomes

$$A_T^{\exp} \approx 4\mathrm{Re}[\epsilon] - 2\mathrm{Re}[x_-] - 2\mathrm{Re}[y], \qquad (45)$$

if the $\Delta S = -\Delta Q$ contributions are also kept. In the CPLEAR experiment, with the proper experimental normalisations, the measured asymptotic asymmetry is [2]:

$$\tilde{A}_T^{\exp} \approx 4\mathrm{Re}[\epsilon] - 4\mathrm{Re}[x_-] - 4\mathrm{Re}[y]. \qquad (46)$$

The average value of $\tilde{A}_T^{\exp}$ was found to be $= (6.6 \pm 1.6) \times 10^{-3}$, which is to be compared to the recent CPLEAR measurement of $(\mathrm{Re}[x_-] + \mathrm{Re}[y]) = (-2 \pm 3) \times 10^{-4}$ [2], indicating that the measured asymmetry is related to the violation of time-reversal invariance. Conclusively, CPLEAR made a direct measurement of time-reversal violation.

One basic point to emphasize here, is that CPLEAR uses only one out of the possible decaying channels, and therefore its measurements are independent of *any unitarity assumption* and the possible existence of invisible decay modes. An interesting question to ask, however, is what information one could obtain on the Kabir asymmetry, from previous measurements plus unitarity [18,9,10]. Unitarity implies the relations

$$\langle K_L^{\text{in}} | K_S^{\text{in}} \rangle = \Sigma_f \langle K_L^{\text{in}} | f^{\text{in}} \rangle \langle f^{\text{out}} | K_S^{\text{in}} \rangle ,$$
$$\langle K_S^{\text{in}} | K_L^{\text{in}} \rangle = \Sigma_f \langle K_S^{\text{in}} | f^{\text{in}} \rangle \langle f^{\text{out}} | K_L \rangle , \qquad (47)$$

where f stands for *all* possible decay channels. Making the additional assumption that the final decay modes satisfy the relation $|f^{\text{in}}\rangle = |f^{\text{out}}\rangle \equiv \langle f^{\text{out}}|^\dagger$ (which is equivalent to making use of *CPT*-invariance of the final state interactions), it is possible to calculate the sum $\langle K_L^{\text{in}} | K_S^{\text{in}} \rangle + \langle K_S^{\text{in}} | K_L^{\text{in}} \rangle$, by *measuring only the branching ratios of kaon decays*. This is what is done in K_L, K_S experiments, where only the *incoming kaon states* are used. In the linear approximation, this sum is equal to $4\,\mathrm{Re}[\epsilon]$ (see Eq. (37)). However, this is an *indirect* determination of *T*-violation, and would not have been possible if invisible decays were present. Also, it would have to be reviewed in stochastic models of *CPT*-violation [19–23]. This is to be contrasted with the results of CPLEAR, which do not rely at all on unitarity and thus on the knowledge of other decay channels than the one used in the analysis.

6. Concluding comments

Motivated by the recent CPLEAR report on the first direct observation of time-reversal non-invariance, we attempted to clarify the situation on measurements of charge conjugation, parity violation and time reversibility, in systems with non-Hermitean Hamiltonians. To do so, we re-discussed the formalism of the neutral kaon system, paying particular attention in the definition of states in the vector space of the system, but also in its dual and in the dual complex spaces. This allows a consistent implementation of the orthogonality conditions for the incoming and outgoing states, used to describe particle-antiparticle mixing and the time evolution of the system.

As a result, we confirm that the asymmetry measured by CPLEAR, is directly related to the definition of *T*-violation. In addition, it does not get affected by time and decay processes. Finally, the experiment uses only one out of the possible decaying channels, therefore its results are independent of any *CPT* or unitarity assumption, and the possible existence of invisible decay modes. We conclude therefore that, CPLEAR indeed made the first direct measurement of *T*-violation.

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