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Upper limit on CP violation in the $B_s^0 - \bar{B}_s^0$ system

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In a previous publication we noted that the time dependence of an incoherent $B^0 - \bar{B}^0$ mixture undergoes a qualitative change when the magnitude of CP violation δ exceeds a critical value. Requiring, on physical grounds, that the system evolve from an initial incoherent state to a final pure state in a monotonic way yields a new upper limit for δ. The recent measurement of the wrong charge semileptonic asymmetry of B_s^0 mesons presented by the D0 collaboration is outside this bound by 1 standard deviation. If this result is confirmed it implies the existence of a new quantum mechanical oscillation phenomenon.

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In a previous paper [1] we studied the time evolution of an incoherent $B^0 - \bar{B}^0$ mixture as a function of the strength of the CP-violating parameter

$$\delta = \langle B_L | B_S \rangle = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2}, \tag{1}$$

where B_L and B_S denote the long-lived and short-lived eigenstates of the system which can be expressed as superpositions of B^0 and $\bar{B}^0$ with the coefficients p, q:

$$\begin{aligned} |B_L\rangle &= \frac{1}{\sqrt{|p|^2 + |q|^2}}(p|B^0\rangle - q|\bar{B}^0\rangle) \\ |B_S\rangle &= \frac{1}{\sqrt{|p|^2 + |q|^2}}(p|B^0\rangle + q|\bar{B}^0\rangle). \end{aligned} \tag{2}$$

The density matrix, in the $B^0 - \bar{B}^0$ basis, was written as

$$\rho(t) = \tfrac{1}{2}N(t)[\mathbb{1} + \vec{\zeta}(t) \cdot \vec{\sigma}], \tag{3}$$

where the normalization function $N(t)$ and the Stokes vector $\vec{\zeta}(t)$ had initial values $N(0) = 1$, $\vec{\zeta}(0) = 0$, respectively. By explicit calculation, one finds

$$\begin{aligned} N(t) = \frac{1}{2(1 - \delta^2)}[&e^{-\gamma_S t} + e^{-\gamma_L t} \\ &- 2\delta^2 e^{-(1/2)(\gamma_S + \gamma_L)t} \cos\Delta m t]. \end{aligned} \tag{4}$$

Furthermore, the magnitude of the Stokes vector is found to have a remarkable relation to $N(t)$:

$$|\vec{\zeta}(t)| = \left[1 - \frac{1}{N(t)^2} e^{-(\gamma_S + \gamma_L)t}\right]^{1/2}. \tag{5}$$

It should be stressed that Eqs. (4) and (5) are valid for any system such as $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$, or $B_s^0 - \bar{B}_s^0$ satisfying the Lee-Oehme-Yang equation of motion [2]. In particular, the relation (5) between $|\vec{\zeta}(t)|$ and $N(t)$ does not involve the parameters δ, $\Delta\gamma$, or Δm, where $\Delta\gamma$ and Δm denote the differences in the decay widths and masses of the eigenstates. Our results are independent of the sign of $\Delta\gamma/\Delta m$.

An upper bound on δ^2 arises from the requirement that the normalization function $N(t)$ should be a monotonically decreasing function of time, i.e. $dN/dt < 0$ for all t. This leads to the constraint

$$\delta^2 \leq \left(\frac{\gamma_S \gamma_L}{(\gamma_S + \gamma_L)^2/4 + \Delta m^2}\right)^{1/2}. \tag{6}$$

In fact, a stronger constraint is obtained by demanding that the time-dependent norm of an arbitrary pure state $\alpha|B^0\rangle + \beta|\bar{B}^0\rangle$ decrease monotonically with time. This bound was obtained by Lee and Wolfenstein [3] and by Bell and Steinberger [4], and is referred to as the unitarity bound:

$$\delta^2_{\text{unit}} \leq \left(\frac{\gamma_S \gamma_L}{(\gamma_S + \gamma_L)^2/4 + \Delta m^2}\right). \tag{7}$$

In [1] we derived a complementary bound from the requirement that the function $|\vec{\zeta}(t)|$ evolve from its initial value $|\vec{\zeta}(0)| = 0$ to its final value $|\vec{\zeta}(\infty)| = 1$ (representing the long-lived pure state B_L) in a monotonic way. This amounts to the assumption that the transformation of the $B^0 - \bar{B}^0$ mixture from its incoherent (high entropy) beginning to its final pure (low entropy) end state is unidirectional in time. The common requirement of monotonicity for $N(t)$ and $|\vec{\zeta}(t)|$ is equivalent to the statement that both of these observables should behave as arrows of time [1].

The monotonicity condition $d|\vec{\zeta}(t)|/dt \geq 0$ can be expressed as a condition on $N(t)$ making use of the relation (5). As shown in [1] this implies an upper bound,

$$\delta^2 \leq \frac{1}{2}\left(\frac{\Delta\gamma}{\Delta m}\right) \sinh\left(\frac{3\pi}{4}\frac{\Delta\gamma}{\Delta m}\right), \tag{8}$$

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which reduces for small $\Delta\gamma/\Delta m$, as found in the $B^0 - \bar{B}^0$ system, to

$$|\delta| \leq \sqrt{\frac{3\pi}{8}} \left| \frac{\Delta\gamma}{\Delta m} \right|. \tag{9}$$

A useful experimental observable is the "wrong charge" semileptonic asymmetry defined (with $q = d, s$) as

$$a_{sl}^q = \frac{\Gamma(\bar{B}_q^0 \to \mu^+ X) - \Gamma(B_q^0 \to \mu^- X)}{\Gamma(\bar{B}_q^0 \to \mu^+ X) + \Gamma(B_q^0 \to \mu^- X)} \tag{10}$$

which is directly related to the *CP* violating parameter (for recent reviews see [5])

$$a_{sl}^q = \frac{1 - |\frac{q}{p}|^4}{1 + |\frac{q}{p}|^4} = \frac{2\delta}{1 + \delta^2} \tag{11}$$

reducing to $a_{sl}^q = 2\delta$ for small δ.

The "coherence bound" for $|a_{sl}^s|$ calculated from (8) is plotted in Fig. 1 versus $|\Delta\gamma/\Delta m|$ together with the unitarity bound obtained from (7). It is immediately seen that for $|\Delta\gamma/\Delta m| \to 0$ the coherence bound is the more effective of the two, providing stringent upper limits.

In a recent analysis of like-sign dimuon pairs in the D0 experiment [6], an asymmetry was measured between $\mu^+\mu^+$ and $\mu^-\mu^-$ final states,

$$A_{sl}^b = \frac{N_b^{++} - N_b^{--}}{N_b^{++} + N_b^{--}} = -(0.957 \pm 0.251 \pm 0.146)\%. \tag{12}$$

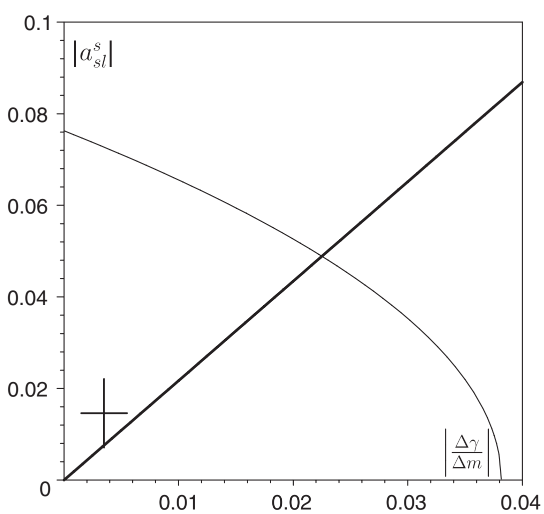

FIG. 1. Constraints on $|a_{sl}^s|$ in the $|a_{sl}^s| - |\Delta\gamma/\Delta m|$ plane resulting from unitarity and monotonicity of $|\zeta(t)|$ for the $B_s^0 - \bar{B}_s^0$ system. The thin line represents the unitarity bound (7) with $\Delta m/\gamma_S = 26.2$ and the thick line our new bound evaluated from (8). The cross represents the experimental result of the D0 experiment for $|a_{sl}^s|$ with the horizontal error bar indicating the uncertainty of $|\Delta\gamma/\Delta m|$.

This asymmetry was determined to be a linear combination of the semileptonic wrong charge asymmetry associated with B_d and B_s mesons:

$$A_{sl}^b = (0.506 \pm 0.043)a_{sl}^d + (0.494 \pm 0.043)a_{sl}^s. \tag{13}$$

Data from B_d -factories [7] provide an estimate of the asymmetry parameter a_{sl}^d:

$$a_{sl}^d = -(0.47 \pm 0.46)\%. \tag{14}$$

Combining this information with Eq. (13), the D0 experiment has extracted a value for the parameter a_{sl}^s which is negative in sign and has the modulus

$$|a_{sl}^s| = (1.46 \pm 0.75)\%. \tag{15}$$

This result is included in Fig. 1. The horizontal error bar reflects the uncertainty in the measured value [8] of $|\Delta\gamma/\Delta m| = 0.0035 \pm 0.002$. The figure shows that the experimental value of $|a_{sl}^s|$ is clearly outside the allowed region violating the coherence bound by about 1 standard deviation.

If the Hamiltonian governing the time dependence of the $B^0 - \bar{B}^0$ system is written as a 2×2 matrix $M_q - \imath\Gamma_q/2$, where M_q and Γ_q are Hermitian, the origin of *CP* violation resides in the phase

$$\Phi_q = \arg\left(-\frac{M_q^{12}}{\Gamma_q^{12}}\right). \tag{16}$$

In terms of this phase, the wrong charge asymmetry parameter is given by

$$a_{sl}^q = \frac{\Delta\gamma}{\Delta m} \tan\Phi_q \tag{17}$$

neglecting higher orders in $\Delta\gamma/\Delta m$. The linear dependence of (17) on $\Delta\gamma/\Delta m$ allows an immediate combination with (9) leading to the constraint

$$|\tan\Phi_s| \leq \sqrt{\frac{3\pi}{2}} = 2.17. \tag{18}$$

Note that the standard model value for Φ_s is very small [9] $\Phi_s^{SM} = 0.24°$. The bound (18) may serve as a new constraint on theories that go beyond the standard model (see, for example, the papers in [10]). It should be stressed that variations of Φ_s must in any case respect the unitarity limit shown in Fig. 1. The upper limit $a_{sl}^s < 7.6\%$ implies the unitarity bound $|\tan\Phi_s| < 22$.

Our analysis based on elementary quantum mechanical reasoning (requiring no assumption about the Hamiltonian other than *CPT* invariance) shows interesting new bounds on the *CP* violating parameters of the $B^0 - \bar{B}^0$ system. It is, however, not granted that nature respects the monotonicity bound following from (8). In this case the D0 result—if true—implies the existence of a new type of quantum mechanical oscillation.

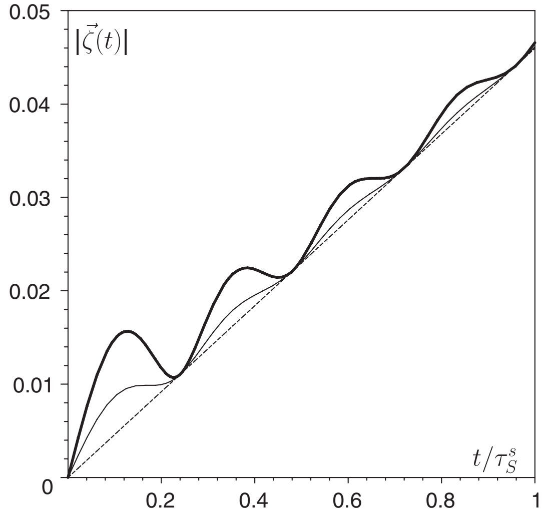

FIG. 2. Plot of $|\vec{\zeta}(t)|$ versus t in units of the lifetime τ_S^s. The thick line is calculated for the nominal D0 value of $\delta = 0.0073$. The strictly monotonic dashed line is obtained for the standard model value of δ. The thin line represents the behavior of $|\vec{\zeta}(t)|$ at the critical value $\delta_{\rm crit} = 0.003\,811\,3$; i.e., the transition between monotonic and nonmonotonic regimes.

To exhibit this new phenomenon we examine in Fig. 2 the characteristics of $|\vec{\zeta}(t)|$ as a function of time in units of τ_S^s, the lifetime of the short-lived B_s^0 meson. The thick line calculated for the nominal D0 value of $\delta = 0.0073$ exhibits two clear oscillations within the first half of the B_s^0 lifetime. By contrast, the dashed line evaluated for the standard model with $\delta \approx 10^{-5}$ is strictly monotonic and in the chosen range of t/τ_S^s even linearly increasing in time. At the critical value $\delta_{\rm crit} = 0.003\,811\,3$ (found by numerical evaluation of the condition $d|\zeta(t)|/dt \geq 0$), the evolution of $|\vec{\zeta}(t)|$ in Fig. 2 follows the middle curve which marks the line of transition between the monotonic and nonmonotonic regimes.

If the D0 result is confirmed it should be possible, in principle, to insert precise data on $N(t)$ into (5) in order to reveal the coherence oscillations shown in the upper curve of Fig. 2. Since the whole effect is induced by the term proportional to δ^2 in (4), the precision required would be at least $\mathcal{O}(10^{-4})$, which may be beyond reach. It is interesting, nevertheless, that precise knowledge of $N(t)$ would allow one to probe the coherence function $\zeta(t)$, and determine whether or not the evolution of coherence in a $B_s^0 - \bar{B}_s^0$ system follows an arrow of time. The D0 asymmetry, taken at face value, suggests that this arrow is broken.

Note added in proof.—Since submission of this paper the Heavy Flavor Averaging Group has reanalyzed [11] the available published [6,12,13] and unpublished [14] experimental data related to a_{sl}^s and extracted an average value of $a_{sl}^s = (-0.85 \pm 0.58)\%$. The central value is much closer to our limit. In view of the large error a precise determination of a_{sl}^s is eagerly awaited.

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