ELECTROMAGNETIC FORM FACTORS OF THE PROTON BETWEEN 15 AND 50 fm^{-2} *

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The external beam of the 2.5 GeV-electron-synchrotron has been used to measure elastic electron proton scattering at four-momentum-transfers between 15 and 50 fm^{-2}. By combining these results with measurements at small angles at DESY, we have obtained the electric and magnetic form factors separately. Their ratio shows a deviation from the scaling law.

The determination of the electric and magnetic form factors of the proton depends on the knowledge of two parameters defining the Rosenbluth straight line. Recent measurements of electron-proton scattering at external electron beams [1] give very good agreement between different groups at small angles, if one allows for a normalization error up to 5%. At larger angles, the measurements up to now have bigger statistical errors and show systematic deviations. We have used the external electron beam of the 2.5 GeV-electron-synchrotron of the University of Bonn to measure cross sections between 30° and 110° for momentum transfers from 15 fm^{-2} to 50 fm^{-2} with statistical errors of 2 to 5%.

To get an intuitive picture of the importance of measurements at various angles, we are using a new plot of the Rosenbluth straight line. Usually, the ratio $R = (\mathrm{d}\sigma/\mathrm{d}\Omega)/(\mathrm{d}\sigma/\mathrm{d}\Omega)_{\mathrm{NS}}$ is plotted for constant q^2 as a function of $\mathrm{tg}^2\,\frac{1}{2}\theta$

$$R = a + b\,\mathrm{tg}^2\,\tfrac{1}{2}\theta$$

with $a = (G_{\mathrm{E}}^2 + \tau G_{\mathrm{M}}^2)/(1+\tau)$, $b = 2\tau G_{\mathrm{M}}^2$ and $\tau = q^2/4M^2$.

We multiply R by $\cos^2\frac{1}{2}\theta$ which we call $\widetilde{R}$, and get $\widetilde{R}$ as a function of $\cos\theta$

$$\widetilde{R} = \tfrac{1}{2}\,(a+b) + \tfrac{1}{2}\,(a-b)\cos\theta$$

The intercept at $\theta_{\mathrm{M}} = 180^{\circ}$ gives directly the magnetic form factor, whereas the intercept at $\cos\theta_{\mathrm{E}} = (3+2\tau)/(1+2\tau)$ depends only on the electric form factor $\widetilde{R}(\theta_{\mathrm{E}}) = 2G_{\mathrm{E}}^2/(1+2\tau)$.

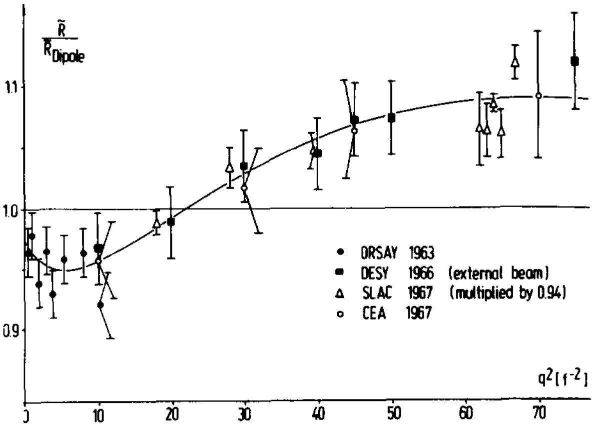

Fig.1. $\widetilde{R}/\widetilde{R}_{\mathrm{dipole}}$ versus q^2. The solid curve is a best fit by eye.

The general features of the experimental arrangement are as follows. The slowly ejected electron beam is focused on a liquid hydrogen target of 5 cm diameter by an achromatic beam transport system. The beam intensity is monitored by a Faraday-cup. The average intensity was about 0.1 μA with a duty cycle of 1%. The beam energy was known within 0.5% and was checked by several devices.

The scattered electrons are analyzed by a magnetic spectrometer consisting of three quadrupoles and one horizontal deflecting magnet. The first quadrupole produces a horizontal angle focus at the collimator which defines the aperture in the horizontal plane independent of the target size. The vertical aperture is defined by the same collimator. The following two quadrupoles and the bending magnet produce a horizontally dispersed image of the collimator at the first counter behind the magnet. This counter is a hodoscope consisting of three scintillation counters. The

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Table 1
Cross-sections. The quoted errors are only random errors. A normalization error of ± 3.5% has to be added.

q^2 [fm^{-2}]	θ [$^\circ$]	E_0 [GeV]	$\frac{d\sigma}{d\Omega}$ [10^{-34} cm^2]
15	30.24	1.629	612.9 ± 6.6
	32.7	1.522	492.6 ± 5.0
	44.48	1.171	240.9 ± 3.5
	51.96	1.042	167.5 ± 3.3
	64.17	0.892	100.3 ± 2.0
	90.08	0.718	45.48 ± 1.0
	110.13	0.647	29.48 ± 0.68
20	32.7	1.789	233.1 ± 3.1
	44.48	1.392	114.3 ± 2.1
	64.17	1.064	45.21 ± 1.1
	90.08	0.865	21.50 ± 0.46
	110.13	0.784	14.05 ± 0.29
25	40.21	1.718	79.36 ± 1.7
	64.17	1.224	24.50 ± 0.55
	90.08	1.003	11.18 ± 0.26
	110.13	0.915	7.027 ± 0.23
30	40.32	1.91	44.88 ± 1.8
	64.17	1.375	14.28 ± 0.30
	90.08	1.135	6.295 ± 0.13
	110.13	1.04	4.194 ± 0.21
40	64.17	1.661	5.485 ± 0.19
	90.13	1.389	2.245 ± 0.091
	110.13	1.282	1.513 ± 0.048
50	90.13	1.632	0.941 ± 0.047

other scintillation counters serve for rejection of background. The electrons are identified by a 1.5 m long freon-filled gas Čerenkov counter. The whole arrangement is mounted on a turntable which can be pivoted around the target in the angular range between 15° and 120°. The spectrometer has an angular acceptance of 0.507 ± 2.0% mster and a momentum resolution of 1.76% for a single hodoscope counter.

The measured cross sections are listed in table 1. For evaluation of the cross section we corrected the counting rates for radiative corrections and real bremsstrahlung before and after scattering. The radiative correction was applied using the formula given by Meister and Yennie [2] and the real bremsstrahlung according to Heitler [3]. Both corrections together were between 20 and 22 percent at all points. The correction due to ionization losses was negligible. The quoted errors come from counting statistics, uncertainties in measuring the scattering angle (< 0.5%) and the energy of the incoming beam (< 0.5%). We have to add a normalization error which we estimate to be about 3.5%. This error is mainly caused by the error in the determination of the aperture (2.0%), the calibration error of the energy (2%), the beam monitoring error (1%), and the uncertainty in the assumed value of the liquid hydrogen density (1%) which was 0.070 g/cm^3.

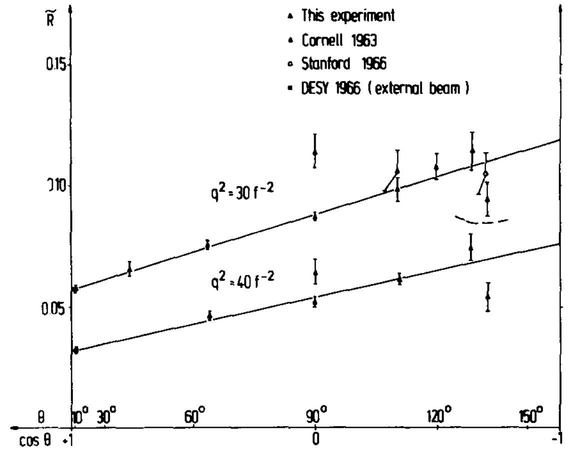

Fig. 2. $\widetilde{R}$ versus cos θ for q^2 = 30 and 40 fm^{-2}.

To get the electric and the magnetic form factors we have to combine our data with the results

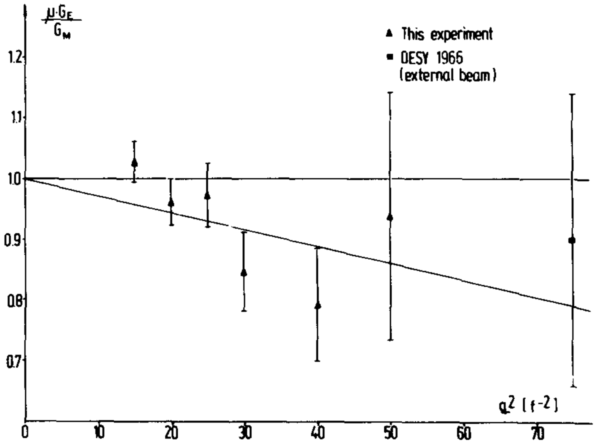

Fig. 3. $G_E\mu/G_M$ versus q^2. The straight line is given by $G_E\mu/G_M = 1 - d \cdot q^2$ with $d = (0.0026 \pm 0.0009)$ fm^2.

Table 2

Form factors. The shown errors are only random errors. In case of G_E and G_M, a normalization error of ± 1.7% has to be added.

q^2 [fm^{-2}]	G_E	G_M/μ	$\mu G_E/G_M$
15	0.3026 ± 0.0064	0.2941 ± 0.0036	1.029 ± 0.034
20	0.2207 ± 0.0068	0.2292 ± 0.0025	0.963 ± 0.039
25	0.1768 ± 0.0076	0.1818 ± 0.0024	0.973 ± 0.054
30	0.1284 ± 0.0083	0.1515 ± 0.0019	0.847 ± 0.064
40	0.0836 ± 0.0085	0.1053 ± 0.0016	0.794 ± 0.091
50	0.070 ± 0.012	0.0746 ± 0.0027	0.938 ± 0.20

of other groups at small angles. We are only using external beam measurements. The most extensive measurements in our momentum-transfer region come from DESY [4]. Comparison within Orsay [5] and CEA [6] data show agreement within an normalization error of 1%. The SLAC data [7] have the same relative behaviour but a difference in normalization of 6% which is inside the claimed uncertainties. Fig. 1 shows the ratios $\tilde{R}$ devided by $\tilde{R}_{\text{dipole}}$, where the dipole fit $G = (1 + q^2/0.71)^{-2}$ has been used. At small angles $\tilde{R}/\tilde{R}_{\text{dipole}}$ is equal a/a_{dipole}. We have drawn a curve by hand which we believe is certain to about 1%. The normalization factor of our points is determined by comparing the Rosenbluth straight line at $q^2 = 15$ fm^{-2}. The deviation of a from the value in fig. 1 is (+ 0.1 ± 1)%. Together with the uncertainty of the curve in fig. 1, we get a normalization error of ± 1.5% compared to the DESY data.

Fig. 2 shows two examples of straight line fits to our data using the a value from fig. 1. The experimental cross sections agree within the statistical errors with a Rosenbluth straight line. There seems to be no deviation due to processes with more than one photon exchange. For comparison, the measurements of other groups are drawn, which have not been used for our fits. The values of G_E and G_M/μ are given in table 2. Fig. 3 shows the ratio $G_E\mu/G_M$ which is independent of a normalization error. The points do not agree with the scaling law. If we assume $G_E\mu/G_M = 1 - d \cdot q^2$, we get

$$d = (0.0026 \pm 0.0009)\ \text{fm}^2 ,$$

which is three times the standard deviation. Since it is very hard, to estimate the systematic errors affecting this result, we believe that further investigations are necessary.

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